TABLE OF CONTENTS

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TABLE OF CONTENTS

A RAPID PERTURBATION PROCEDURE FOR DETERMINING NONLINEAR FLOW SOLUTIONS: APPLICATION TO TRANSONIC TURBOMACHINERY FLOWS

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SUMMARY

An investigation was conducted to develop perturbation procedures and associated computational codes for determining nonlinear flow solutions, with the objective of establishing a method for minimizing computational requirements associated with parametric studies of transonic flows in turbomachines. The theoretical analysis involved the development of a rapid method for calculating first-order changes in nonlinear flow solutions due to variations of an arbitrary geometrical or flow parameter.

The procedure developed and evaluated, referred to as the direct correction method, was found to be capable of determining highly accurate approximations to families of strongly nonlinear solutions which are either continuous or discontinuous, and which represent variations in some arbitrary parameter. The method consists of defining a unit perturbation by employing two nonlinear solutions which differ from one another by a nominal change in some geometric or flow parameter, and then using that unit perturbation to predict a family of related nonlinear solutions over a range of parameter variation. Coordinate straining is used in determining the unit perturbation to account for the movement of discontinuities and maxima of high-gradient regions due to the perturbation. While simultaneous multiple-parameter perturbations can be treated by the method, the theoretical development and results presented in this initial study are for the single-parameter perturbation problem.

Although the procedure is generally applicable, the results reported here have been directed toward nonlinear aerodynamic applications. Attention is focused in particular on transonic

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flows which are strongly supercritical and exhibit large surface shock movement over the parametric range studied; and on subsonic flows which display large pressure variations in the stagnation and peak suction pressure regions. Flows past both isolated airfoils and compressor cascades involving a wide variety of flow and geometry parameter changes are reported. Comparisons with the corresponding 'exact' nonlinear solutions indicate a remarkable accuracy and range of validity of such a procedure. Computational time of the method, beyond the determination of the base solutions, is trivial.

1. INTRODUCTION

Given the remarkable growth in capability of advanced computational methods for the determination of a spectrum of nonlinear phenomena in such diverse disciplines as fluid dynamics, structures, and nuclear physics to name just a few - a capability which has already made many difficult calculations routine and which is certain to improve in the future - it is apparent that a need exists for complementary methods capable of alleviating, at least in part, the usage limitations imposed on these methods by their run times. The need becomes particularly compelling when large numbers of related cases are required as in parametric or design studies. Techniques such as direct acceleration procedures provide an important means of reducing computer time by improving computational efficiency of the solution algorithm, but these and similar methods, which enhance the solution algorithm itself, represent only a partial answer. What is most desirable is a means to minimize the actual number of separate calculations required in a particular application by extending, over some parametric range, the usefulness of each individual solution determined by these computationally expensive procedures.

Consequently, the basic motivation underlying this study is to extend the usefulness of such numerical solutions computed for specific turbomachinery configurations and flow conditions with a view toward reducing the computational requirements now necessary. The nature of the present investigation is both exploratory and developmental in the sense that aspects of the procedure such as validity, range of application, and economy will be investigated, and a computational code embodying all the results of the study will be developed.

Two fundamental methods for accomplishing such a perturbation procedure are available: a classical approach involving posing and solving linear perturbation equations; and a direct correction method employing two or more nonlinear base solutions. In this report, both of these methods are discussed; and an evaluation of the latter method, based on a large number of different applications, is made.

A crucial aspect of such perturbation methods is their ability to accurately treat regions where either discontinuities or high gradients exist. For the results presented here coordinate straining is introduced as a means of accounting properly for the displacement of discontinuities due to an arbitrary change in some solution parameter. This is shown to result in highly accurate perturbation predictions in the vicinity of the discontinuity. That idea has also been extended to improve predictions in the vicinity of other high-gradient regions.

Although the procedures developed are generally applicable, the specific results reported here are for aerodynamic applications. Since one of the primary objectives of this study was to provide a definitive proof-of-concept of such a perturbation method, a large variety of perturbation results based on transonic small-disturbance and full potential solutions were studied and are presented for nonlinear subsonic and transonic flows past both isolated airfoils and compressor cascades. In order to enable a critical evaluation of the range of validity and accuracy of the straining procedure, emphasis was placed on transonic flows which are strongly supercritical and exhibit large surface shock movement over the parametric range studied; and on subsonic flows which display large pressure variations in the stagnation and peak suction pressure regions.

2. ANALYSIS

2.1 Perturbation Concept and Methods

The basic hypothesis underlying the present procedure is that a range of solutions in the vicinity of a previously determined or base solution can be calculated to first-order accuracy in the incremental change of the varied parameter by determining a linearized unit perturbation solution Q_p defined according to the relation

$$\underbrace{Q}_{\substack{\text{Approximate solution for}\\ \text{conditions differing from}\\ \text{those of the base solution}\\ \text{by an amount characterized}\\ \text{by } \varepsilon}} = \underbrace{Q_o}_{\substack{\text{Base solution}\\ \text{for some flow}\\ \text{quantity } Q}} + \underbrace{\varepsilon \cdot \{Q_p\}}_{\substack{\text{Linearized perturbation}\\ \text{solution for a unit}\\ \text{change of } \varepsilon}} \tag{1}$$

The effectiveness of such a method, of course, depends upon the ability of the relationship defined by equation (1) to remain accurate over a range ε of practical significance, and the fact that the unit perturbation Q_p need be determined only once. The significance of the unit perturbation Q_p is obvious. It represents the local rate of change of the base flow solution Q_o with respect to the particular quantity, say q, perturbed; that is $Q_p = (\partial Q/\partial p)_o$.

Two generic methods exist for determining Q_p, each differing in philosophy and having its own particular strengths and weaknesses. We refer to these methods simply as the linear perturbation equation method and the direct correction method.

The linear perturbation equation method represents the classical approach for performing a perturbation analysis and proceeds by establishing and solving a linear differential equation for the perturbation. Although in the present application, we confine out interest solely to the first-order term, the complete procedure represents a rational approximation scheme capable of continuation to any order. The method proceeds by expanding the dependent variables in an ascending power series in the incremental change ε of the varied parameter, inserting that representation into the full governing equations and then assembling the result into a corresponding series of linear equations in ascending orders in ε. Higher-order solutions

in general depend on both base flow plus lower-order solutions. Determination of the appropriate boundary conditions is done in a similar fashion.

The power of the linear perturbation equation method is that it requires the calculation of only one nonlinear base solution. With that information, any number of individual perturbations can then be calculated, subject to the particular governing linear partial differential equations and boundary conditions which apply. The disadvantages are that each perturbation problem must be posed individually, including differential equations and boundary conditions. Furthermore, it may be necessary to simplify the governing equations and boundary conditions to a point where they can be solved rapidly relative to rerunning the base flow procedure. Moreover, the perturbation solutions themselves may be quite sensitive to the base flow solutions which usually enter into the perturbation problem through the differential equation and sometimes through the boundary conditions as well.

The fundamental limitation of the method is the restriction of the range over which the perturbation procedure remains valid to a linear one. Since this characteristic depends upon the local behavior of the base flow with respect to the varied parameter, no general statement regarding range of validity is possible. Typical behavior for a given class of flows must be ascertained by checks with the base flow procedure. Initially unknown at the outset of an application with this technique, then, are the accuracy requirements imposed on the base solution by the perturbation procedure and the range of parameter variation over which the linear assumption is valid.

For the alternative method, the perturbation solution per unit change of the varied parameter, Q_p, is determined simply by differencing two nonlinear base flow solutions removed from one another by some nominal change of a particular flow or geometrical quantity. A unit perturbation solution is then obtained by dividing that result by the change in the perturbed quantity. Related solutions are determined by multiplying the unit perturbation by the desired parameter change and adding that result to the base flow solution. This simple procedure, however, only works directly for continuous flows for which the perturbation change does not alter the solution domain. For those perturbations which change the flow domain, coordinate stretching (usually obvious) is necessary to insure proper definition of the unit perturbation solution. Similarly, for discontinuous flows, coordinate straining is necessary to account for movement of discontinuities due to the perturbation solution.

The attractiveness of the correction method is that it is not restricted to a linear variation range but rather replaces the nonlinear variation between two base solutions with a linear

fit. This de-emphasizes the dependence and sensitivity inherent in the linear perturbation equation method on the local rate of change of the base flow solution with respect to the varied quantity. For many applications, particularly at transonic speeds, the flow is highly sensitive, and the linear range of parameter variation can be sufficiently small to be of no practical use. Furthermore, other than the approximation of a linear fit between two nonlinear base solutions, the direct correction method is not restricted by further approximations with respect to the governing differential equations and boundary conditions. Rather, it retains the full character of the original methods used to calculate the base flow solutions. Most importantly, no perturbation differential equations have to be posed and solved, only algebraic ones. In fact, it isn't even necessary to know the exact form of the perturbation equation, only that it can be obtained by some systematic procedure and that the perturbations thus defined will behave in some 'generally appropriate' fashion so as to permit a logical perturbation analysis. For situations involving perturbations of physical parameters, such as reported here, the governing perturbation equations are usually transparent, or at least readily derivable. Finally, in applying this method it isn't necessary to work with primitive variables; rather the procedure can be applied directly to the final quantity desired.

The primary disadvantage of this method is that two base solutions are required for each parameter perturbation considered. Furthermore, both flows must be topologically similar, i.e., discontinuities or other characteristic features must be present in both base solutions used to establish the unit perturbation.

2.2 Previous Applications

Detailed studies of the linear perturbation equation method to sensitive transonic flows, with a view toward testing the method as an effective tool for reducing computational requirements, have not been done. The primary reason is that such studies quickly become overwhelming. Each perturbation problem must be posed individually, subject to its own particular governing equations and boundary conditions; and then a separate computational code for the perturbation established. Generally, the governing equations and boundary conditions of the perturbation, even though they are linear, are more involved than those for the base solution. Additionally, the computational and convergence characteristics can pose similar or additional problems from those of the base flow procedure.

In an attempt to examine some of these problems for transonic applications in at least a preliminary fashion, an application of the linear perturbation equation method to

transonic turbomachinery flows was made in reference 1. The conclusions obtained from that study were that reasonable results could be anticipated from the method for blade geometry changes, such as blade thickness and angle of attack. Less satisfactory results were obtained for perturbation changes in overall quantities, such as blade spacing and free-stream Mach number, a result that could be anticipated a priori since such perturbations alter the basic character of the flow more rapidly. The most significant conclusion of that study was the demonstration of the primary limitation of the linear perturbation equation method. That is, for sensitive flows such as occur in transonic situations, the basic linear variation assumption fundamental to the technique is sufficiently restrictive that the permissible range of parameter variation becomes so small as to be of limited practical use. Some preliminary applications of the direct correction method, however, displayed a significantly wider range of perturbation solution validity, in particular for strongly supercritical flows when coordinate straining was employed to account for shock movement.

2.3 Coordinate Straining

The concept of employing coordinate straining to remove nonuniformities from perturbation solutions of nonlinear problems is well established and originally suggested by Lighthill (ref. 2) three decades ago. The basic idea of the technique is that a straightforward perturbation solution may possess the appropriate form, but not quite at the appropriate location. The procedure is to strain slightly the coordinates by expanding them as well as the dependent variables in an asymptotic series. It is often unnecessary to actually solve for the straining. It can generally be established by inspection. The final uniformly valid solution is then found in implicit form, with the strained coordinate appearing as a parameter.

In the original applications of the method (ref. 3), it was applied in the 'classical' sense; that is, series expansions of the dependent and independent variables in ascending powers in some small parameter were inserted into the full governing equation and boundary conditions, and the individual terms of the series determined. An ingenious variation in the application of the method was made by Pritulo (ref. 4) who demonstrated that if a perturbation solution in unstrained coordinates has been determined and found to be nonuniform, the coordinate straining required to render that solution uniformily valid can be found by employing straining directly in the known non-uniform solution, and then solving algebraic rather than differential equations. The idea of introducing strained coordinates a posteriori has since been applied to a variety of different problems (see ref. 3), and forms the basis of the current applications.

The fundamental idea underlying coordinate straining as it relates to the application of perturbation methods to supercritical transonic flows is illustrated geometrically in figure 1. In the upper plot on the left, two typical transonic pressure distributions are shown for a highly supercritical flow about a nonlifting symmetric profile. The distributions can be regarded as related nonlinear flow solutions separated by a nominal change in some geometric or flow parameter. The shaded area between the solutions represents the perturbation result that would be obtained by directly differencing the two solutions. We observe that the perturbation so obtained is small everywhere except in the region between the two shock waves, where it is fully as large as the base solutions themselves. This clearly invalidates the perturbation technique in that region and most probably somewhat ahead and behind it as well. The key idea of a procedure for correcting this, pointed out by Nixon (refs. 5,6), is first to strain the coordinates of one of the two solutions in such a fashion that the shock waves align, as shown in the upper plot on the right of figure 1, and then determine the unit perturbation. Equivalently, this can be considered as maintaining the shock wave location invariant during the perturbation process, and assures that the unit perturbation remains small both at and in the vicinity of the shock wave. Obviously, shock points are only one of a number of characteristic high-gradient locations such as stagnation points, maximum suction pressure points, etc., in which the accuracy of the perturbation solution can degrade rapidly. The plots in the lower left part of the figure 1 indicate such a situation and display typical transonic pressure distributions which contain multiple shocks and high-gradient regions. Simultaneously straining at all these locations, as indicated in the lower right plot, serves to minimize the unit perturbation over the entire domain considered, and provides the key to maximizing the range of validity of the perturbation method.

2.4 Theoretical Formulation for Single-Parameter Perturbations

In order to provide the theoretical essentials of the correction method, consider the formulation of the procedure at the level of the full potential equation, as most of the results presented here are based on that level. We denote the operator L acting on the velocity potential Φ as that which results in the two-dimensional full potential equation for Φ, i.e.

$$L[\Phi] = 0 \qquad (2)$$

If we now expand the potential in terms of zero- and higher-order components in order to account for the variation of some arbitrary geometrical or flow parameter q

$$\Phi = \Phi_o + \varepsilon\Phi_1 + \ldots$$
$$q = q_o + \Delta q \tag{3}$$

and then insert this into the governing equation (2), expand the result, order the equations into zero- and first-order components, and make the obvious choice of expansion parameter $\varepsilon = \Delta q$, we obtain the following governing equations for the zero- and first-order components

$$L[\Phi_o] = 0$$
$$L_1[\Phi_1] + \frac{\partial}{\partial q} L[\Phi_o] = 0 \tag{4}$$

Here L_1 is a linear operator whose coefficients depend on zero-order quantities and $\partial L[\Phi_o]/\partial q$ represents a 'forcing'term due to the perturbation. Actual forms of L_1 and the 'forcing' term are provided in reference 1 for a variety of flow and geometry parameter perturbations of a two-dimensional turbomachine, and in reference 7 for profile shape perturbations of an isolated airfoil. An important point regarding equation (4) for the first-order perturbation Φ_1 is that the equation represents a unit perturbation independent of the actual value of the perturbation quantity ε.

Appropriate account of the movement of discontinuities and maxima of high-gradient regions due to the perturbation is now accomplished by the introduction of strained coordinates (s,t) in the form

$$x = s + \varepsilon x_1(s,t)$$
$$y = t + \varepsilon y_1(s,t) \tag{5}$$

where

$$x_1(s,t) = \sum_{i=1}^{N} \delta x_i x_{1_i}(s,t)$$
$$y_1(s,t) = \sum_{i=1}^{N} \delta y_i y_{1_i}(s,t) \tag{6}$$

and $\varepsilon\delta x_i$, $\varepsilon\delta y_i$ represents individual displacements of the N strained points, and $x_{1_i}(s,t)$, $y_{1_i}(s,t)$ are straining functions associated with each of the N strained points. Introducing the strained coordinate equations (5) and (6) into the expansion formulation leaves the zero-order result in equation (4) unchanged, but results in a change of the following form for the perturbation

$$L_1[\phi_1] + L_2[\phi_o] + \frac{\partial}{\partial q} L[\phi_o] = 0 \tag{7}$$

Here the operators are understood to be expressed in terms of the strained (s,t) coordinates, and the additional operator L_2 arises specifically from displacement of the strained points. In references 6 and 7, specific expressions for L_2 are provided for selected perturbations involving transonic small-disturbance and full potential equation formulations. The primary point, however, with regard to perturbation equation (7) expressed in strained coordinates is that it remains valid as before for a unit perturbation and independent of ε.

In employing the correction method, equation (7) for the unit perturbation is solved by taking the difference between two solutions obtained by the full nonlinear procedure after appropriately straining the coordinates. If we designate the two solutions for some arbitrary flow quantity Q as base Q_o and calibration Q_c, respectively, of the varied parameter, we have for the predicted flow at some new parameter value q (ref. 8)

$$Q(x,y) = Q_o(s,t) + \frac{\varepsilon}{\varepsilon_o} [Q_c(\bar{x},\bar{y}) - Q_o(s,t)] \tag{8}$$

where

$$\begin{aligned} \bar{x} &= s + \varepsilon_o x_1(s,t) \\ \bar{y} &= t + \varepsilon_o y_1(s,t) \\ x &= s + \frac{\varepsilon}{\varepsilon_o} [\bar{x} - s] \\ y &= t + \frac{\varepsilon}{\varepsilon_o} [\bar{y} - t] \\ \varepsilon_o &= q_c - q_o \\ \varepsilon &= q - q_o \end{aligned} \tag{9}$$

In the following section, applications of the correction procedure are made to predict surface properties. Also provided are the particular forms of the straining functions equation (6) for those applications.

2.5 Current Applications: Surface Pressures

For the current applications, we have employed coordinate straining with the correction method to predict surface pressure distributions for a wide variety of single-parameter geometrical flow perturbations of isolated airfoils and cascades. In that instance where flow properties are required along some contour, the solutions can be represented by

$$\begin{aligned} Q(x;\varepsilon) &\sim Q_o(s) + \varepsilon Q_1(s) + \ldots \\ x &\sim s + \varepsilon x_1(s) + \ldots \end{aligned} \tag{10}$$

where x is the independent variable measuring distance along the contour or a convenient projection of that distance, s is the strained coordinate, and ε a small parameter representing the change in some flow or geometrical variable which we wish to vary.

In order to determine the first-order corrections $Q_1(s)$, we require a base and calibration solution in which the calibration solution is determined by varying an arbitrary parameter q by some nominal amount from the base flow value.

In this way, the first-order correction $Q_1(s)$ can be determined as

$$Q_1(s) = \frac{Q_c(\bar{x}) - Q_o(s)}{q_c - q_o} \tag{11}$$

where Q_c is the calibration solution corresponding to changing the parameter q to a new value q_c, $\bar{x}$ is the strained coordinate pertaining to the Q_c calibration solution, and $q_c - q_o$ represents the change in the q parameter from its base flow value. If we now desire to keep invariant during the perturbation process a total of N points corresponding to discontinuities or high-gradient maxima, we can represent the solution by:

$$Q(x;\varepsilon) = Q_o(s) + \varepsilon Q_1(s) \tag{12}$$

where

$$Q_1(s) = \frac{Q_c(\bar{x}) - Q_o(s)}{\varepsilon_c} \tag{13}$$

$$\bar{x} = s + \sum_{i=1}^{N} \varepsilon_c(\delta x_i^c)\cdot x_{1_i}(s) \tag{14}$$

$$x = s + \sum_{i=1}^{N} \varepsilon(\delta x_i^c)\cdot x_{1_i}(s) \tag{15}$$

$$\varepsilon_c = q_c - q_o \tag{16}$$

$$\varepsilon = q - q_o \tag{17}$$

$$\varepsilon_c(\delta x_i^c) = (x_i^c - x_i^o) \tag{18}$$

$$\varepsilon(\delta x_i^o) = \frac{\varepsilon}{\varepsilon_c}(x_1 - x_i^o) \tag{19}$$

Here $\varepsilon_c(\delta x_i^c)$ given in equation (18) represents the displacement of the ith invariant point in the calibration solution from its base flow location due to the selected change ε_c in the q parameter given by equation (16), $\varepsilon(\delta x_i^c)$ given in equation (19) represents the predicted displacement of the ith invariant point from its base flow location due to the desired change ε in the q parameter given by equation (17), and $x_{1_i}(s)$ is a unit-order straining function having the property that

$$x_{1_i}(x_k^o) = \begin{cases} 1 & k = i \\ 0 & k \neq i \end{cases} \tag{20}$$

which assures alignment of the ith invariant point between the base and calibration solutions.

In addition to the single condition equation (20) on the straining function, it may be convenient or necessary to impose additional conditions at other locations along the contour. For example, it is usually necessary to hold invariant the end points along the contour, as well as to require that the straining vanish in a particular fashion in those locations. All of these conditions, however, do not serve to determine the straining uniquely. The nonuniqueness of the straining, nevertheless, can often be turned to advantage, either by selecting particularly simple classes of straining functions or by requiring the straining to satisfy further constraints convenient for a particular application. An example of the effect of employing two different straining functions for a strongly-supercritical flow was

provided in reference 6. Here we provide additional results demonstrating some of the limitations of various polynomial straining functions and provide some comparisons with piecewise-continuous functions. The particular classes of straining functions employed were continuous polynomial and linear piecewise-continuous. For these two classes, the functional forms of the straining can be compactly written. For example, equation (14) becomes, for continuous polynomial straining

$$\bar{x} = s + \sum_{i=2}^{N-1} L_i(s) \cdot (x_i^c - x_i^o) \tag{21}$$

where L_i are Lagrangian coefficients given by

$$L_i(s) = \prod_{\substack{k=1 \\ k \neq i}}^{N} \frac{(s - x_k^o)}{(x_i^o - x_k^o)} \tag{22}$$

whereas for linear piecewise-continuous straining, $\bar{x}$ is given by

$$\bar{x} = s + \sum_{i=2}^{N-1} \left\{ \frac{x_{i+1}^o - s}{x_{i+1}^o - x_i^o} \cdot (x_i^c - x_i^o) + \frac{s - x_i^o}{x_{i+1}^o - x_i^o} \cdot (x_{i+1}^c - x_{i+1}^o) \right\} H(x_{i+1}^o - s) \cdot H(s - x_i^o) \tag{23}$$

where H denotes the Heaviside step function. As discussed above, it is usually necessary to hold invariant both of the end points along the contour in addition to the points corresponding to discontinuities or high-gradient maxima. Consequently, for the results reported here, the array of invariant points in the base and calibration solutions have been taken as

$$\begin{aligned} x_i^o &= \{0,\ x_1^o,\ x_2^o,\ \ldots.,\ x_n^o,\ 1\} \\ x_i^c &\quad \{0,\ x_1^c,\ x_2^c,\ \ldots.,\ x_n^c,\ 1\} \end{aligned} \tag{24}$$

where the contour length has been normalized to unity. Figure 2 provides a summary of the various combinations of flows and straining functions employed.

3. RESULTS

One of the primary objectives of the present investigation is to explore the accuracy and range of validity of such perturbation procedures to determine to what extent they are capable of providing results useful in an engineering analysis. To this end, we have tested the correction method with coordinate straining over a wide variety of different geometrical and flow condition perturbations, including applications to both isolated airfoils and compressor cascades. In particular, since the ability of the method to account accurately for the movement of discontinuities and maxima of high-gradient but continuous regions is essential if such procedures are to be of general use, emphasis was placed on transonic flows which are strongly supercritical and exhibit large surface shock movement over the parametric range studied. Base flow theoretical solutions were determined from small-disturbance transonic potential (ref. 9) and full potential solutions (refs. 10, 11, 12). In the results to follow, which were selected as typical from systematic calculations of a much larger number of cases, the choice of base and calibration solutions was often made at the limits of validity of the procedure to observe how well the method works under such conditions.

3.1 Perturbation Results for Supercritical Single-Shock Flows and Subcritical Flows

3.1.1 Supercritical applications.- In figure 3, we present results for a thickness-ratio perturbation of strongly supercritical flows past a nonlifting cascade of biconvex profiles at $M_\infty = 0.80$ having a spacing-to-chord ratio of $H/C = 1.0$. The dotted and dashed results on the figure represent the base and calibration surface pressure distribution for $\tau = (0.075, 0.065)$, respectively, and were obtained by solving the transonic small-disturbance potential equation using the code TSFOIL (ref. 9). An x-grid having 48 points on the blade profile was used. These solutions were then used to determine the unit perturbation. The open circles represent the perturbation solution for $\tau = 0.073$ in the plot on the left and for $\tau = 0.070$ in the plot on the right. Those perturbation results are meant to be compared with the solid lines in the plots which are the corresponding nonlinear solutions obtained by rerunning TSFOIL at the new thickness ratios. Quadratic straining was used with shock point and leading and trailing edges held invariant. The base and calibration flow shock-point locations for this example, as well as for all of the supercritical cases presented here, were determined as the point where the pressure coefficient passed through critical with compressive gradient.

With regard to the results, several points are noteworthy. Selection of a cascade rather than an isolated airfoil provides a more sensitive transonic flow situation. Additionally, the choice of a highly supercritical base and almost subcritical calibration solution provides both an example of extreme separation between the two nonlinear solutions used to define the unit perturbation, as well as a situation where one solution is near the limits of validity of the perturbation analysis. Recall that both solutions must be topographically similar, i.e., must contain the same number of discontinuities (shocks) and other characteristic features.

We note that comparisons of the perturbation results with the nonlinear calculations are very satisfactory for both thickness ratios, with the only discrepancy being a slight disagreement at the lower thickness ratio ($\tau = 0.070$) at several points in the post-shock region. Additional calculations not presented here in which a more reasonable choice of calibration solution is made, say at $\tau = 0.070$, removes that discrepancy as well. The main point provided by the results of figure 3 is that for certain classes of supercritical flows even widely separated base solutions can be used to provide reasonable perturbation predictions.

In figure 4, we provide similar strongly supercritical results again for interpolation-only perturbation solutions, but in this instance on a somewhat finer grid. These results employed full potential base solutions (ref. 10), and represent thickness ratio perturbations of nonlifting symmetric free-air flows past NACA four-digit thickness-only airfoils at $M_\infty = 0.820$. The body-fitted mesh employed had 75 points on both upper and lower surfaces, which is half again as many as in the preceding example. For the base and calibration flows, the thickness ratios were $\tau = 0.120$ and 0.080, respectively. Comparisons between the perturbation predictions and the full nonlinear calculation are exhibited in figure 4 for $\tau = 0.110$, 0.105, 0.100, and 0.095. We note that the comparisons are remarkably good, in particular, in the region of the shock. The first-order perturbation accurately predicts both shock location and the post-shock expansion behavior. Reference to the coarser grid results given in figure 3 indicates that the finer grid resolution clearly enhances the perturbation result, indicating that better accuracy and a larger range of validity of the perturbation solutions can be anticipated when fine-grid base solutions are used to define the unit perturbation.

In the two preceding examples, perturbation results were provided for interpolation-only between widely spaced base and calibration solutions. In figure 5, we provide similar strongly supercritical thickness-ratio perturbation results for extreme solution extrapolation using very closely spaced base and calibration solutions (ref. 10). The upper plots display results

for extrapolation downward from base and calibration flows past nonlifting NACA 00XX profiles with $\tau = 0.115$ and 0.120 at $M_\infty = 0.820$. Perturbation predictions are shown for $\tau = 0.105$ and 0.100, which represent $\Delta\tau$ excursions from the base flow ($\tau = 0.115$) that are two and three times the parameter change between the base and calibration solutions ($\Delta\tau = 0.005$) used to define the unit perturbation. For these results, comparisons with the full nonlinear calculations are very good. The lower plots display similar results for extreme extrapolation upward from base and calibration solutions have τ = 0.095 and 0.090. Perturbation predictions are shown for $\tau = 0.105$ and 0.110, which again represent excursions from the base flow that are two and three times the parameter change between the base and calibration solutions. In this instance, while comparisons of the perturbation results and the full nonlinear solutions for both cases are good, the results at τ = 0.110 are beginning to display some not surprising discrepancies near the shock wave, indicating that the perturbation result is nearing the limit of its range of validity for this particular choice of base and calibration flows.

The results indicated in figure 5, however, clearly demonstrate that not only is accurate solution extrapolation possible, but that for some situations even closely spaced nonlinear solutions can be used to cover a wide range of related solutions. Additionally, the range of parameter variation in this example over which the perturbation results remain accurate - i.e., parameter changes three times the difference between the two nonlinear solutions used to define the unit perturbation - is remarkable, and far beyond what one would anticipate for a first-order correction.

Perturbation results using a more reasonable choice of base and calibration solutions are provided in figure 6. Those results involve Mach number perturbations of highly supercritical full potential (ref. 10) flows past a NACA 0012 airfoil at $\alpha = 0°$. The base and calibration results are for $M_\infty = 0.800$ and 0.820, and the comparisons indicated are for perturbation results interpolated to $M_\infty = 0.810$ and extrapolated downward to $M_\infty = 0.790$. As in the case of the geometric perturbations given in figures 4 and 5, these perturbation results are also in very good agreement with the nonlinear calculations at the new Mach numbers. For this perturbation, as well as for a number of other Mach number perturbations, we have separately determined the perturbation result in two different ways. First, we have taken cognizance of the fact that a Mach number perturbation alters the governing differential equation for the first-order perturbation from that of other geometric or flow parameter changes; and have used the suggestion of reference 6 to consider such perturbations via a transonic small-disturbance approximation, whereby the same perturbation equation can be preserved by employing a modified expansion parameter ε. An alternative procedure is to treat a

Mach perturbation directly and interpret ε as the difference in Mach number. We have done these calculations and compared the perturbation results for a number of cases using both full potential solutions, as for the results shown in figure 6, and transonic small-disturbance solutions, and have observed no essential difference between the two sets of results. The perturbation results presented in figure 6 correspond to those for ε equal to the difference in Mach number.

All of the supercritical perturbation results presented in figures 3 to 6 have been for symmetric flows and have employed a quadratic straining function. In figure 7, we present results for an angle of attack perturbation of lifting flows past a NACA 0012 profile at $M_\infty = 0.70$. The full potential (ref. 10) base and calibration solutions are at $\alpha = 3.0°$ and $4.0°$, with comparisons of the perturbation and full nonlinear results shown for $\alpha = 3.5°$ and $2.5°$. Cubic straining has been used with the invariant points corresponding to the lower trailing edge, stagnation point, shock point, and the upper trailing edge (see fig. 2). We note that $\alpha = 3.5°$, the perturbation results are very good everywhere, in particular, in the vicinity of the shock and stagnation regions. At $\alpha = 2.5°$, the perturbation results are still very good in the shock and stagnation regions and on most of the upper and lower surface, but near the trailing edge a discrepancy has occurred. The cause of this discrepancy lies solely with the cubic straining function used. It is due to the fact that although the straining vanishes identically at the trailing edge, for the particular choice of base and calibration solutions in this example, the straining in the near vicinity of the trailing edge becomes sufficiently large to introduce a misalignment in the unit perturbation in that high-gradient region. The correction to this is discussed in the section describing piecewise-continuous straining functions.

3.1.2 Subcritical applications.- Although supercritical flows are clearly of central concern in any transonic analysis for which the perturbation methods presented here would be used, applications to subcritical nonlinear flows are also of significance. To this end, we have applied these same techniques to a variety of subcritical flows to examine their accuracy and range of validity for such applications.

In figure 8, we present some summary results for four different subcritical perturbation applications to an isolated airfoil. All of these results are based on full potential solutions (ref. 10) with quadratic straining holding invariant the stagnation point and the trailing edge points. The plot on the upper left displays comparisons for a camber line perturbation of a lifting flow with $M_\infty = 0.50$ and $\alpha = 2°$ past an airfoil having a NACA 0012 thickness distribution and a parabolic-arc camber line having the maximum camber located at midchord. Base

and calibration flows with camber ratio $h/c = 0.02$ and 0.01 were used to extrapolate perturbation results to $h/c = 0.05$. Comparisons with the full result is essentially exact. The plot on the upper right provides similar results for a thickness-ratio perturbation of a lifting flow with $M_\infty = 0.50$ and $\alpha = 2.0°$ past NACA 00XX thickness-only airfoils. Base and calibration flows with $\tau = 0.12$ and 0.04 were used to provide interpolation results at $\tau = 0.08$. Again, the agreement is essentially exact even in the peak suction pressure region. The plot on the lower left provides angle-of-attack perturbation results for $M_\infty = 0.50$ flow past a NACA 0012 airfoil, using base/calibration results for $\alpha = 4.0°$, $2.0°$ to predict results at $\alpha = 3.0°$, with the agreement again being quite good. The final comparisons given in the plot on the lower left are for a Mach number perturbation of a lifting flow at $\alpha = 2°$ past an airfoil having a NACA 0012 thickness distribution and a parabolic-arc camber line with camber ratio $h/c = 0.03$ at midchord. Base/calibration results at $M_\infty = 0.40$, 0.60 were used to predict results at $M_\infty = 0.55$, with good agreement with the full nonlinear calculation.

In figure 9, we present similar summary results for subcritical perturbation applications to a compressor cascade having a 4% biconvex thickness distribution and a 1% parabolic-arc camber line blade, a pitch of $t/c = 0.37$, and oncoming Mach number $M_\infty = 0.770$. These results are based on the full potential solution procedure of reference 11 and have also used quadratic straining to hold the trailing edge points and stagnation point invariant. The plots in the upper part of the figure represent an inflow angle perturbation, with base/calibration inflow angles $\beta_i = 47.8°$, $49.8°$ used to predict extrapolation results in the plot on the left for $\beta_i = 48.8°$ and interpolation results in the plot on the right for $\beta_i = 48.8°$. In the lower left plot, interpolation results are displayed for an outflow angle perturbation with base/calibration outflow angles $\beta_o = 31.5°$, $39.5°$ used to predict the flow at $\beta_o = 35.5°$. The lower right plot provides interpolation results for a rotational speed perturbation with base/calibration rotational speeds $\omega = 967{,}667$ rad/sec used to predict the flow at $\omega = 827$ rad/sec. In all of these results, the perturbation results are good, including the regions near the leading and trailing edge where a peaky behavior due to local grid resolution is observed.

3.2 Comparison of Continuous and Piecewise-Continuous Straining Function Perturbation Results

The results presented in figures 10 to 13 illustrate the effect of using different straining functions to determine the perturbation results. Comparisons are provided for several strongly supercritical flows, demonstrating the differences in perturbation solutions between using quadratic and cubic straining

functions and corresponding piecewise-continuous straining functions.

Figure 10 displays a comparison for a symmetric supercritical thickness-ratio perturbation at $\tau = 0.110$ for which results based on quadratic straining were given in figure 4. In that figure the open circles denote the previously obtained perturbation results using quadratic straining, while the asterisks denote the corresponding result when using linear piecewise-continuous straining. The points held invariant are the leading and trailing edges and the shock point. For this case there is virtually exact agreement everywhere between the two perturbation results as well as with the nonlinear result. An analogous comparison with a cubic straining result is provided in figure 11 where the invariant points are the lower trailing edge, stagnation point, shock point, and upper trailing edge. Displayed in that figure as open circles are the cubic-straining supercritical angle-of-attack perturbation results at $\alpha = 2.5°$ which were previously given in figure 7. Asterisks denote the corresponding linear piecewise-continuous straining perturbation result. We note that the discrepancy near the trailing edge caused by the cubic straining has been effectively removed in the piecewise-continuous result. Moreover, the good agreement with the full nonlinear result which the cubic result displayed near the shock and stagnation regions, as well as over the remainder of the airfoil surface, is also obtained with the piecewise-continuous result.

Finally, we have found that when employing quadratic, cubic, and higher-order polynomials as straining functions, for certain combinations of base flow shock location and shock movement between base and calibration solutions, particularly when large shock movements are involved, the polynomial straining functions will strain some points off the airfoil surface. This, of course, invalidates the determination of the unit perturbation, and requires that a different straining function be employed. Piecewise-continuous straining functions provide a simple means of avoiding such difficulties.

In figures 12 and 13, we have provided examples illustrating this effect for both quadratic and cubic straining functions. Figure 12 provides a comparison of perturbation results obtained using quadratic (open circles) and linear piecewise-continuous (asterisks) straining applied to a supercritical Mach number perturbation for symmetric nonlifting flow past a NACA 0012 airfoil. Widely separated base/calibration flows (ref. 10) at $M_\infty = 0.820$ and 0.750 were used to predict the flow at $M_\infty = 0.810$. The spurious behavior near the leading edge displayed by the open circles is due to the quadratic function moving points in the strained calibration solution off the airfoil surface. The piecewise-continuous results indicated by the asterisks display a smooth variation in that region, and provide good agreement

everywhere with the full nonlinear result. Figure 13 provides a corresponding comparison for cubic straining. Angle-of-attack perturbation results at $M_\infty = 0.70$ for flow past a NACA 0012 profile using base/calibration results (ref. 10) at $\alpha = 2.25°$ and $4.00°$ are used to predict the flow at $\alpha = 3.25°$. The unusual results displayed by the open symbols near the trailing edge indicate that the cubic function has strained points off the airfoil surface in that region. However, the linear piecewise-continuous result corrects that problem and displays good agreement with the nonlinear calculation in that region as well as at the shock and stagnation point.

3.3 Perturbation Applications to Complex Supercritical Flows

In order to provide a severe test of the perturbation procedure, we have applied the method to a number of transonic flows that are characterized by surface pressure distributions having multiple shock and/or high-gradient locations, such as those typified schematically in the lower plots of figure 1. Demonstration of the ability of the perturbation method to predict accurately such classes of flows, which are typical of those encountered in certain transonic turbomachinery applications, is crucial to the present study. In order to accomplish such a demonstration, we have investigated two separate classes of sensitive supercritical transonic flows, i.e. those with multiple-shock waves, and those having a single shock together with multiple high-gradient regions. Examples of perturbation results for such flows are provided below.

3.3.1 Multi-Shock Supercritical Flows.- In figure 14, we present results for an angle-of-attack perturbation of supercritical lifting flows past a NACA 0012 profile at $M_\infty = 0.80$. These highly sensitive flows exhibit two shocks, one on each the upper and lower surface. The full potential (ref. 10) base and calibration flows employed are at $\alpha = 0.50°$ and $0.20°$, with comparisons of the perturbation and full nonlinear results shown for $\alpha = 0.0°$, $0.1°$, $0.4°$, and $0.6°$. Piecewise-continuous linear straining has been used with the invariant points corresponding to the lower trailing edge, lower surface shock point, stagnation point, upper surface shock point, and upper trailing edge (see fig. 2). We note that the symmetrical extrapolation result at $\alpha = 0.0°$ is separately predicted from both the upper surface and lower surface pressure distributions, and, as can be seen, the results are quite good. The remaining results at $\alpha = 0.1°$, $0.4°$, and $0.6°$, which represent both extrapolation and interpolation from the base and calibration flows, are in excellent agreement with the full nonlinear result. As an indication of the sensitivity of these flows, we have found that the lower surface shock

disappears at an angle of attack of approximately 0.8°; yet the lower surface pressure distribution is well predicted by the perturbation result over the parametric range studied.

3.3.2 Supercritical Compressor Cascade Flows.- As an example of the ability of the method to predict a complex supercritical flow, in figure 15 we provide results for oncoming Mach number perturbation of supercritical flows past a cascade composed of Jose Sanz (ref. 12) profiles. For these results, the oncoming and exit flow angles are 30.81° and 0.09°, respectively, the blade twist is 9.33°, while the gap to chord ratio is 1.028. The full potential (ref. 12) base and calibration flow oncoming Mach numbers are $M_\infty = 0.77$ and 0.81, with comparisons of perturbation and full nonlinear results shown at $M_\infty = 0.75$, 0.79, 0.89, and 0.83. Piecewise-continuous linear straining was employed with invariant points at the lower trailing-edge, stagnation point, shock point and upper trailing edge. As with the multiple-shock example shown in figure 14, we note that the perturbation predictions are in excellent agreement with the nonlinear results. In particular, we note that the perturbation procedure captures the variation of the plateau-like pressure distribution on the upper surface near the leading edge, the location and strength of the shock, the post-shock expansion region, the rapid expansion near the trailing edge, and the expansion on the lower surface near the stagnation point, indicating a capability for treating very general flow situations.

4. CONCLUSIONS AND RECOMMENDATIONS

An evaluation has been made of a perturbation procedure for determining highly accurate approximations to families of nonlinear solutions which are either continuous or discontinuous, and which represent variations in some arbitrary parameter. The procedure employs a unit perturbation, determined from two nonlinear solutions which differ from one another by a nominal change in some geometric or flow parameter, to predict a family or related nonlinear solutions. Coordinate straining is used in determining the unit perturbation in order to account properly for the motion of discontinuities and maxima of high-gradient regions. Extensive perturbation calculations based on full potential nonlinear solutions have been carried out. These calculations cover a variety of flow and goemetric parameter perturbations involving isolated airfoils and compressor cascades at both subsonic and transonic flow conditions. Particular emphasis was placed on supercritical transonic flows which exhibit large surface shock movements over the parameter range studied; and on subsonic flows which display large pressure variations in the stagnation and peak suction pressure regions. Perturbation results for single-parameter perturbations, characterized by both extreme solution interpolation using widely separated base flow solutions and extreme solution extrapolation using closely spaced based flow solutions, were obtained in order to determine the accuracy and range of validity of the method. Additionally, calculation of perturbation results were made to investigate the effectiveness of employing piecewise-continuous straining functions rather than polynomial (quadratic, cubic, quartic) functions. Multi-shock and other complex flow situations were studied in order to examine the capability for treating general transonic flows.

Comparisons of the perturbation results with the corresponding 'exact' nonlinear solutions indicate a remarkable accuracy and range of validity of the perturbation method across the spectrum of examples reported. Geometry and flow parameter perturbations are treatable with equal success. Solution interpolation and extrapolation are both feasible. Results evaluating the polynomial and piecewise-continuous straining functions indicate that the piecewise-continuous functions are superior. The latter class of straining functions eliminate both the problem of unwanted straining in the domain of interest, as well as the problem of spurious straining out of the domain. Finally, it was demonstrated that this procedure can successfully treat flows containing multiple shocks and high-gradient regions by simultaneously straining all of these characteristic points. Computational time of the method, beyond the determination of the base solutions, is trivial. A code encompassing these developments has been written for the single-parameter perturbation problem

and is included as part of this report. Based on these results, we conclude that such a perturbation procedure can provide a means for substantially reducing computational requirements in design studies or other applications where large numbers of related nonlinear solutions are needed. Further development is needed, however, to provide a computational tool of wide utility. Because of the practical need in design or parametric studies to consider variations in several parameters simultaneously, we suggest the development of the capability for multiple-parameter perturbations, making full use of the current developments of the single parameter procedure. That procedure should incorporate a limiting-parameter calculation whereby the parameter bounds with respect to each varied parameter are determined. Finally, in order to demonstrate their ultimate power and utility, these procedures should now be tested by actual application to a practical problem which involves the high-frequency use of expensive computational codes in order to determine a large number of related flow solutions. We suggest transonic turbomachinery blade design optimization studies as both feasible and of high current importance.

APPENDIX A - USER'S MANUAL FOR COMPUTER PROGRAM PERTURB

A.1 INTRODUCTION

The purpose of this appendix is to describe the operation of the computer code which was developed in conjunction with the theoretical work presented in this report, and to provide sufficient detail to permit convenient use and change of the program. The program computes and plots an arbitrary flow variable on a contour surface by employing the strained-coordinate perturbation method previously discussed. The plot package included in this version refers to system routines at the Stanford University Center for Information Processing facility. In general, the plotting software must be supplied by the user according to the requirements of his operating system. This can be accomplished directly by replacing or modifying the subroutines PLOT, LIMITS, and ROUND.

A description of the general operating procedure of the program is given, together with complete description of both input and output. The program is written in FORTRAN IV and has been developed on an IBM 3033 computer. Typical run times are 1 to 3 seconds. The storage requirements are $50K_{10}$.

A.2 PROGRAM DESCRIPTION

The program calculates both continuous and discontinuous nonlinear perturbation solutions which represent a single-parameter change in either geometry or flow conditions by employing a strained-coordinate procedure. The method utilizes a unit perturbation, determined from two previously calculated solutions ('base' and 'calibration' solutions) obtained from an 'expensive' computational procedure and displaced from one another by some reasonable change in geometry or flow variable, to predict new nonlinear solutions over a range of parameter variation.

This version of the procedure is configured to predict and plot an arbitrary flow variable (e.g., pressure coefficient) on the surface of a blade or airfoil, and can account for the motion of:

1. one or more critical points (shock points),

2. a stagnation point,

3. a maximum-suction-pressure point,

or simultaneously for any combination of these.

The program is also configured to compare the perturbation-predicted solutions with the corresponding 'exact' solutions obtained by employing the same 'expensive' computational procedure used to determine the base and calibration solutions.

The coordinate straining employed is piecewise linear with the end points and up to six interior points held invariant. At the option of the user, these additional interior points may be arbitrarily preselected, or chosen from among the minimum, maximum, and critical points automatically located by the program itself.

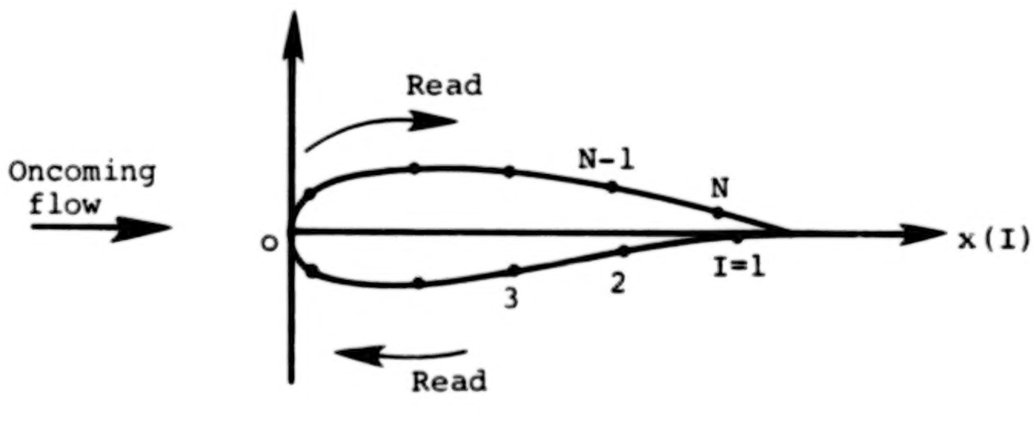
Critical or shock points are located on the basis of a user-supplied statement function defining the critical value of the dependent variable as a function of some single flow variable. The program default is with dependent variable y defined as pressure coefficient, with the independent variable being Mach number. In this case, the critical value is defined as

$$y_{crit} = C_p^* = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{2 + (\gamma - 1)M_\infty^2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}} - 1\right] \qquad (A-1)$$

where γ is the ratio of specific heats. If instead of surface pressure coefficient, the surface velocity distribution were used, then the value of y_{crit} would be given by

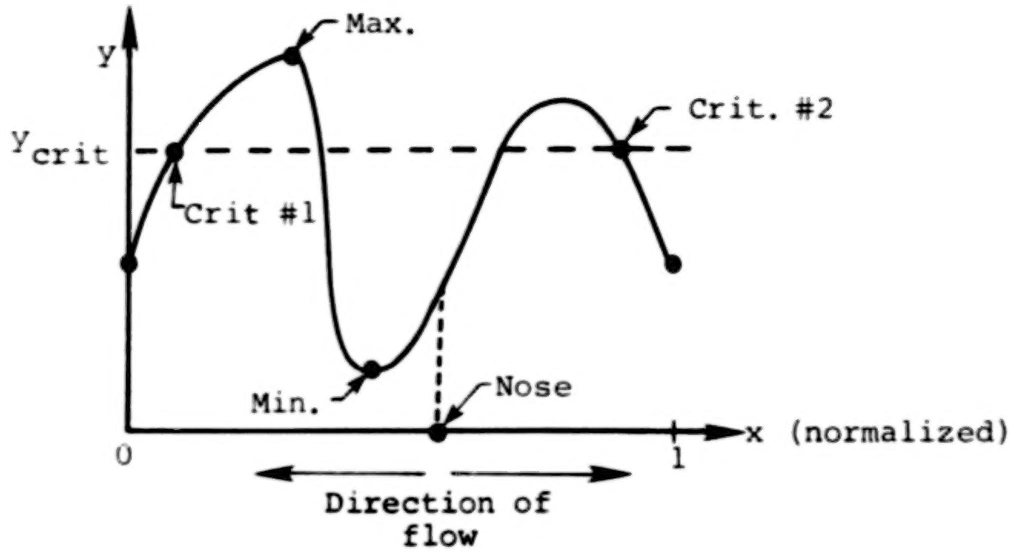
$$y_{crit} = \frac{V^*}{V_\infty} = \left(\frac{\gamma + 1}{2 + (\gamma - 1)M_\infty^2}\right)^{\frac{1}{\gamma-1}} . \qquad (A-2)$$

Data for base, calibration, and comparison solutions (if available) are input as an array x(I) of coordinates and a corresponding array y(I) giving the dependent variable at each coordinate location, where $1 \leq I \leq N$ and $N \leq 200$.



The leading edge is at x = 0; the data are read in beginning on the lower surface at the point farthest from the leading edge and proceeding clockwise around the surface as shown in the sketch. Data for the different solutions need not correspond to identical locations on the surface, except for the initial and final points, i.e., x(1) and x(N) must be the same for all cases. The program normalizes the x coordinates ($0 \leq x \leq 1$) such that x = 0 corresponds to I = 1 and x = 1 to I = $\overline{N}$.

The base and calibration solutions are searched for minimum, maximum, and critical points, e.g.,

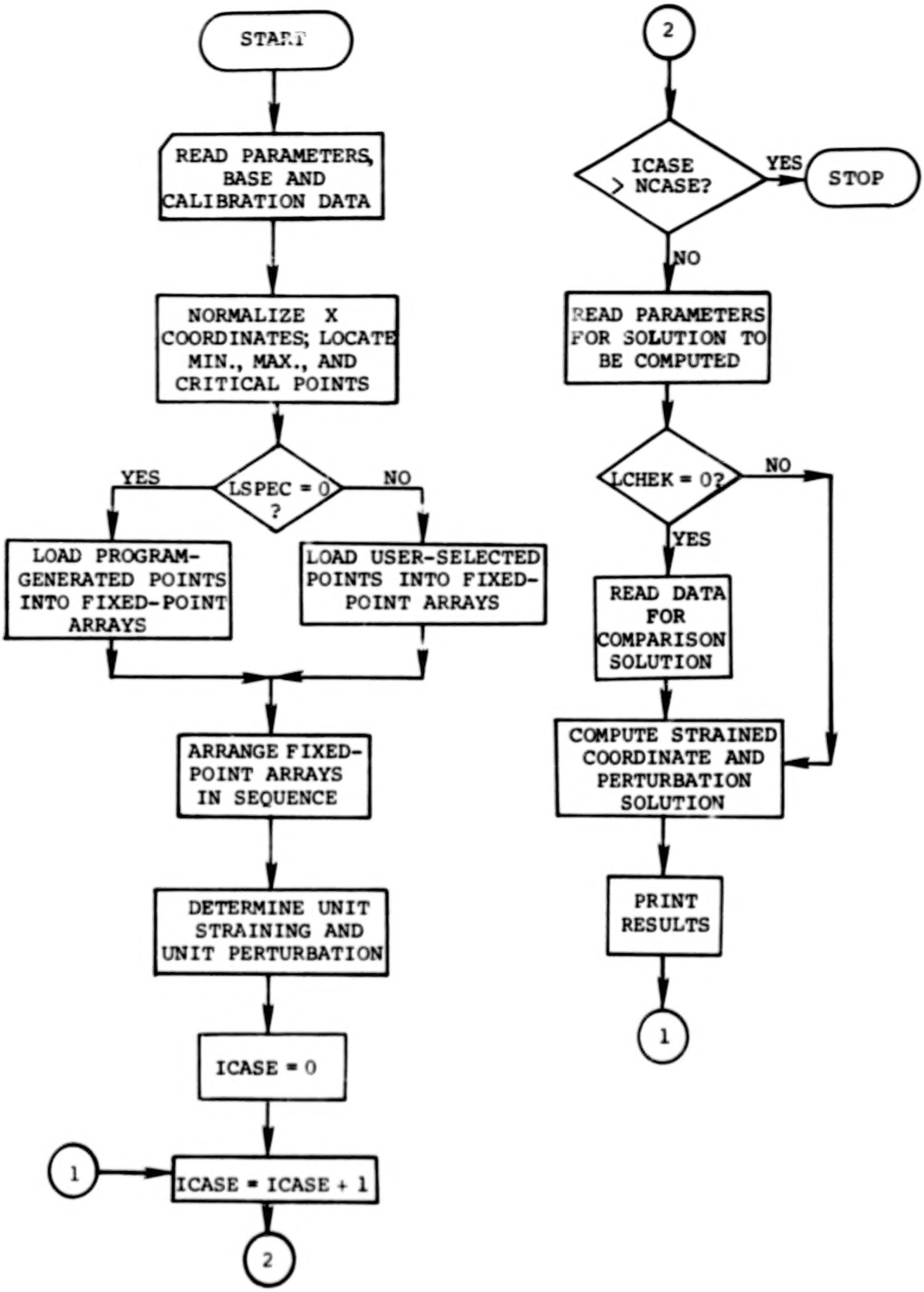


Note that the sign of dy/dx in <u>physical</u> coordinates is used in determining the critical points. For example, both critical points indicated on the above figure correspond to dy/dx > 0 in physical coordinates, since at point #1 the physical coordinate increases in the direction from right to left, whereas at point #2 it increases from left to right.

The points to be held invariant in straining are either selected from among those located by the program or individually specified by the user, after which the unit coordinate straining and unit perturbation are computed.

Data for the test cases is then read in and nonlinear perturbation solutions constructed from the unit perturbation.

A.3 PROGRAM FLOW CHART



A.4 DICTIONARY OF INPUT VARIABLES

A — Scaling parameter in straining procedure. A = -x(1), where x(1) is location of first data point on lower surface (see PROGRAM DESCRIPTION).

B — Scaling parameter in straining procedure. B = x(N), where x(N) is location of last data point on upper surface (see PROGRAM DESCRIPTION).

LCHEK — Specifies whether or not perturbation solution is to be compared with an exact solution.

LCHEK = 0 ... no comparison
LCHEK = 1 ... comparison

LECHO — Controls whether or not input deck is printed.

LECHO = 0 ... no print
LECHO = 1 ... print

LOC0(I) — Array of length 6 containing subscripts of user-specified invariant points in base solution; operational only when LSPEC = 1.

LOC1(I) — Array of length 6 containing subscripts of user-specified invariant points in calibration solution; operational only when LSPEC = 1.

LPERT — Specifies type of perturbation; operational only when LCHEK = 1 and only affects output from plot subroutine.

LPERT = 1 ... thickness-ratio perturbation
LPERT = 2 ... angle-of-attack perturbation
LPERT = 3 ... Mach-number perturbation

LSELCT(I) — Array of length 6 of which NSELCT elements are read in; operational only when LSPEC = 0, and specifies nature of points to be held invariant according to the code:

1 ... minimum point held invariant
2 ... maximum point held invariant
3 ... 1st critical point held invariant
4 ... 2nd critical point held invariant
5 ... 3rd critical point held invariant
6 ... 4th critical point held invariant

Note that critical point ordering is determined from order of occurrence starting at the lower surface at the point furthest from the leading edge and proceeding clockwise around the surface (see PROGRAM DESCRIPTION).

Note that the code numbers can be assigned in any order, e.g.,

LSELCT(1) = 1		LSELCT(1) = 4
LSELCT(2) = 3	and	LSELCT(2) = 1
LSELCT(3) = 4		LSELCT(3) = 3

are equivalent, both corresponding to NSELCT = 3, with the minimum, and first and second critical points held invariant.

LSPEC Controls how invariant points in straining are specified.

LSPEC = 0 ... invariant points selected from among those located by the program, using the array LSELCT(I)

LSPEC = 1 ... invariant points preselected by user, using the arrays LOC0(I), LOC1(I)

LUNIT Controls whether or not unit coordinate straining and unit perturbation are printed.

LUNIT = 0 ... no print
LUNIT = 1 ... print

M0,M1,M2 Oncoming Mach numbers in base, calibration, and perturbation solutions.

N Number of locations for which data are input for base, calibration, and comparison solutions.

NAME Character string of length 2 which symbolizes dependent variable, e.g., "CP" for pressure coefficient.

NCASE Number of cases for which perturbation solutions are to be computed.

NSELCT Number of points (in addition to end points) to be held invariant in straining; note: $1 \leq$ NSELCT ≤ 6.

Q0,Q1,Q2 Values of perturbation parameter in base, calibration, and perturbation solutions.

TITLE Character string of length 80; identifies job and is printed as headline on first page of output.

XBASE(I),XCALB(I),XCHEK(I)...

Arrays of surface coordinates in base, calibration, and comparison solutions.

YBASE(I),YCALB(I),YCHEK(I)...

Arrays of dependent variables in base, calibration, and comparison solutions.

A.5 PREPARATION OF INPUT DATA

A.5.1 Description of Input

Item 1 One card, containing the parameters N, NCASE, LSPEC, LECHO, LUNIT, LCHEK, LPERT.

Item 2 One card, containing either

(a) NSELCT, (LSELCT(I), I=1,NSELCT)
(b) NSELCT, (LOC0(I), I=1,NSELCT),
(LOC1(I), I=1,NSELCT)

where (a) and (b) correspond to LSPEC = 0 and LSPEC = 1, respectively.

Item 3 One card, containing the character string TITLE.

Item 4 One card, containing the character string NAME.

Item 5 One card, containing the scaling parameters A and B.

Item 6 One card, containing M0(real) and Q0.

Item 7 One set of K cards, where K = 1 + INT(N/8), containing data for x coordinate in base solution.

Item 8 One set of K cards, K as above, containing data for dependent variable in base solution.

Item 9 One card, containing M1(real) and Q1.

Item 10 One set of K cards, K as above, containing data for x coordinate in calibration solution.

Item 11 One set of K cards, K as above, containing data for dependent variable in calibration solution.

Item 12 One card, containing M2(real) and Q2.

Item 13 One set of K cards, K as above, containing data for x coordinate in comparison solution. This item is required only when LCHEK = 1.

Item 14 One set of K cards, K as above, containing data for dependent variable in comparison solution. This item is required only when LCHEK = 1.

Note: Items 12-14 are required, in sequence, as many times as specified by NCASE.

A.5.2 Format of Input Data

Item no. 1: 1 card

Variable	N	NCASE	LSPEC	LECHO	LUNIT	LCHEK	LPERT
Card column	5	10	15	20	25	30	35
Format type	I	I	I	I	I	I	I

Item no. 2a (LSPEC = 0): 1 card

Variable	NSELCT	LSELCT(1)	LSELCT(2)	----	LSELCT(NSELCT)		
Card column	5	10	15	20	25	30	35
Format type	I	I	I	I	I		

Item no. 2b (LSPEC = 1): 1 card

Variable	NSELCT	LOC0(1)	----	LOC0(NSELCT)	LOC1(1)	----	LOC1(NSELCT)
Card column	5	10	15	20	25	30	35
Format type	I	I	I	I	I	I	I

Item no. 3: 1 card

Variable	TITLE							
Card column	10	20	30	40	50	60	70	80
Format type	A							

Item no. 4: 1 card

Variable	NAME	
Card column	2	
Format type	A	

Item no. 5: 1 card

Variable	A	B	
Card column	10	20	
Format type	F	F	

BLANK PAGE

BLANK PAGE

Item no. 6: 1 card

Variable	M0	Q0	
Card column	10	20	
Format type	F	F	

Item no. 7: K cards, K = 1 + INT(N/8), 8 values per card

Variable	XBASE(1)	XBASE(2)	XBASE(3)	----				
Card column	10	20	30	40	50	60	70	80
Format type	F	F	F	F	F	F	F	F

Item no. 8: K cards, K as above, 8 values per card

Variable	YBASE(1)	YBASE(2)	YBASE(3)	----				
Card column	10	20	30	40	50	60	70	80
Format type	F	F	F	F	F	F	F	F

Item no. 9: 1 card

Variable	M1	Q1	
Card column	10	20	
Format type	F	F	

Item no. 10: K cards, K as above, 8 values per card

Variable	XCALB(1)	XCALB(2)	XCALB(3)	----				
Card column	10	20	30	40	50	60	70	80
Format type	F	F	F	F	F	F	F	F

Item no. 11: K cards, K as above, 8 values per card

Variable	YCALB(1)	YCALB(2)	YCALB(3)	----				
Card column	10	20	30	40	50	60	70	80
Format type	F	F	F	F	F	F	F	F

Item no. 12: 1 card

Variable	M2	Q2	
Card column	10	20	
Format type	F	F	

Item no. 13: K cards, K = 1 + INT(N/8), 8 values per card

Variable	XCHEK(1)	XCHEK(2)	XCHEK(3)	----				
Card column	10	20	30	40	50	60	70	80
Format type	F	F	F	F	F	F	F	F

Item no. 14: K cards, K as above, 8 values per card

Variable	YCHEK(1)	YCHEK(2)	YCHEK(3)	----				
Card column	10	20	30	40	50	60	70	80
Format type	F	F	F	F	F	F	F	F

A.6 DESCRIPTION OF OUTPUT

The first output item consists of a banner page, and the card images of the input data, the latter only if LECHO = 1.

The second item is a page headed by the job title, listing:

1. the input parameters relevant to the actual calculation;
2. the critical values of the dependent variable;
3. the locations of the minimum, maximum, and critical points found by the program;
4. the straining points selected;
5. the invariant points.

Results for unit straining of XBASE, and unit perturbation of the dependent variable are the third item output; this is done only if LUNIT = 1.

The fourth item (repeated for each case computed) summarizes the results of the calculation. The Mach number, the value of the perturbation parameter, and the critical value of the dependent variable are printed first, followed by the locations of the minimum, maximum, and critical points in the perturbation solution and comparison solution (if any). Then follows a table listing XBASE, YBASE, XCALB, YCALB, XPERT (the strained coordinate), and YPERT (the computed value of the dependent variable). If LCHEK = 1, three additional columns list XCHEK, YCHEK, and YPERT(INT), the latter being interpolated values of YPERT (the computed solution) at the points given by XCHEK. This allows direct numerical comparison of YPERT with YCHEK, since the values of XPERT and XCHEK do not coincide in general.

A.7 ERROR MESSAGES

NUMBER OF CRITICAL POINTS IN
BASE AND CALIBRATION SOLUTIONS
ARE UNEQUAL - CALCULATION ENDED

This message will be printed if critical points are specified in straining (LSPEC = 0) and the number of critical points in base and calibration solutions are unequal. The remedy is to avoid use

of critical points in straining, or to use base and calibration solutions having equal numbers of critical points.

NUMBER OF CRITICAL POINTS
SELECTED EXCEEDS NUMBER
ACTUALLY LOCATED - CALCULATION
ENDED

This message will be printed if more critical points are specified in straining (LSPEC = 0) than the number located by the program. The remedy is to specify a number of points less than or equal to the actual number.

ORDER OF SPECIFIED POINTS IN
BASE AND CALIBRATION SOLUTIONS
DOES NOT CORRESPOND

This message will be printed if the fixed points specified (LSPEC = 0) occur in a different sequence in the base and calibration solutions. The remedy is to use base and calibration solutions having the same qualitative features.

A.8 SAMPLE CASE

The sample case presented in this section provides results (6 perturbation calculations and comparisons with 'exact' nonlinear solutions) for a multiple-shock flow for which partial results were provided in figure 14 of the main text. The calculation is for angle-of-attack perturbations of full potential flows past an isolated NACA 0012 airfoil at $M_\infty = 0.80$. The base and calibration angles-of-attack are $\alpha_b = 0.500°$ and $\alpha_c = 0.200°$. Perturbation results are determined at $\alpha =$ 0.00°, 0.10°, 0.30°, 0.40°, 0.60°, and 0.70° and are compared with previously-calculated 'exact' nonlinear flows at those angles.

The input data is tabulated in figure A.1, with item numbers corresponding to those indentified in Section A.5.1 and A.5.2. The first card, item 0, indicates that there are 149 points (N = 149) at which data will be input for the base, calibration, and comparison solutions; that there will be 5 cases (NCASE = 6) for which perturbation solutions are to be computed, that the invariant points will be located by the program (LSEPC = 0), that the input card deck will not be printed (LECHO = 0), that the information regarding the unit perturbation will be printed (LUNIT = 1), that there will be a comparison of the perturbation results with the exact solution (LCHEK = 1), and that the plot output will denote an angle-of-attack perturbation (LPERT = 2). The second card, item 2a, indicates that there will be three invariant points (NSELCT = 3) in addition to the end points; and that those points will be (1) where the maximum occurs (LSELCT(1) = 2) i.e. the stagnation point, (2) the first critical point (LSELCT(2) = 3) i.e. the 1st shock point found when moving forward on the bottom surface from the trailing edge, and (3) the second critical point (LSELCT(3) = 4) i.e. the 2nd shock point. The next card, item 3, contains the identifying title. On the next card, item 4, the 2 length character string indicates that the dependent variable for print output will be symbolized by a 'CP' denoting pressure coefficient. Item 5 indicates that the coordinates of the data points to be read in will start at x = 1.0 on the upper surface (refer to descriptions in A.4). The next card, item 6, indicates that the base flow values of Mach number and perturbation parameter (angle-of-attack in this case) are MO = 0.80 and QO = 0.50, respectively. The following 19 cards, item 7, provide the 149 base flow values of the surface coordinates, while the next 19 cards, item 8, provide the 149 base flow values of the dependent variable (pressure coefficient). Items 9, 10, and 11 indicate for the calibration flow the corresponding information given by the items 6,7, and 8 for the base flow. Items 12, 13, and 14, of which there are six sets corresponding to the 6 cases to be studied, provide analogous information as items 6,7, and 8, but now refer to the 'exact' nonlinear results. These, of course have been previously computed at the indicated

values of angle-of-attack (Q2) given in Item 12, and are included here for comparative purposed to enable assessment of the perturbation results.

Figure A.2 provides an abbreviated print output for the sample case, while figure A.3 provides the plot output of the results for the six cases, and display the base (••••), calibration (----), perturbation (****), and 'exact' nonlinear (———) flow solutions.

Column
No.

Item No.	5	10	15	20	25	30	35	80
1	149	6	0	0	1	1	2	
2a	3	2	3	4				
3	SAMPLE CASE - ALPHA PERTURBATION FOR MULTI-SHOCK ISOLATED AIRFOIL FLOW							
4	CP							
5	-1.0	1.0						
6	.800000	.500000						
7	.999495	.997758	.995132	.991597	.987091	.981656	.975290	.968014
	.959827	.950749	.940799	.930018	.918425	.906020	.892863	.878994
	.864393	.849140	.833235	.816758	.799690	.782110	.764038	.745536
	.726603	.707279	.687646	.667702	.647510	.627108	.606518	.585801
	.564955	.544062	.523144	.502239	.481369	.460595	.439937	.419417
	.399095	.378972	.359110	.339530	.320253	.301322	.282758	.264582
	.246838	.229547	.212731	.196413	.180616	.165342	.150655	.136558
	.123054	.110166	.097919	.086338	.075427	.065211	.055678	.046895
	.038773	.031422	.024818	.018980	.013897	.009506	.006089	.003379
	.001483	.000362	.000000	.000362	.001483	.003379	.006089	.009506
	.013897	.018980	.024818	.031422	.038773	.046895	.055678	.065211
	.075427	.086338	.097919	.110166	.123054	.136558	.150655	.165342
	.180616	.196413	.212731	.229547	.246838	.264582	.282758	.301322
	.320253	.339530	.359110	.378972	.399095	.419417	.439937	.460595
	.481369	.502239	.523144	.544062	.564955	.585801	.606518	.627108
	.647510	.667702	.687646	.707279	.726603	.745536	.764038	.782110
	.799690	.816758	.833235	.849140	.864393	.878994	.892863	.906020
	.918425	.930018	.940799	.950749	.959827	.968014	.975290	.981656
	.987091	.991597	.995132	.997758	.999495			
8	.454705	.454255	.414440	.401123	.370901	.335399	.303686	.274379
	.247635	.222849	.198568	.174301	.153539	.133375	.110559	.091755
	.072839	.053774	.034598	.016276	-.000869	-.019323	-.037701	-.055942
	-.071935	-.090513	-.109358	-.126685	-.146033	-.164340	-.183752	-.202450
	-.221343	-.241733	-.260634	-.278560	-.295525	-.311898	-.326422	-.330417
	-.403291	-.497623	-.600235	-.668220	-.700287	-.711940	-.712103	-.707273
	-.699075	-.687197	-.672616	-.656143	-.635506	-.613957	-.591617	-.562085
	-.539317	-.496814	-.460480	-.418394	-.372256	-.319692	-.268045	-.205256
	-.131694	-.056739	.037329	.148457	.285927	.443489	.629985	.831392
	1.019305	1.145394	1.161699	1.053470	.849325	.606187	.371494	.168265
	.005374	-.133558	-.243041	-.335771	-.410394	-.483396	-.543403	-.590331
	-.638836	-.679361	-.716673	-.749187	-.779281	-.807499	-.834131	-.855738
	-.876312	-.896774	-.914886	-.932147	-.947948	-.961781	-.974530	-.986034
	-.997187	-1.006749	-1.016031	-1.022717	-1.029553	-1.034871	-1.039277	-1.041815
	-1.041697	-1.039425	-1.033151	-1.019966	-.993560	-.947006	-.857172	-.470554
	-.047162	.060158	.067065	.059619	.053938	.049536	.050777	.055100
	.061931	.069436	.079745	.092114	.105449	.119529	.134216	.153439
	.170576	.188732	.210704	.233000	.256263	.281298	.309273	.339810
	.374275	.403616	.416177	.495153	.494705			
6	.800000	.200000						
7	.998133	.994611	.990123	.984707	.978365	.971116	.962959	.953914
	.944001	.933260	.921710	.909350	.896242	.882424	.867877	.852680
	.836534	.820417	.803411	.785896	.767891	.749457	.730593	.711341
	.691779	.671909	.651791	.631465	.610951	.590310	.569541	.548725
	.527834	.507056	.406263	.465556	.444984	.424539	.404292	.384243
	.364455	.344946	.325740	.306878	.288381	.270271	.252590	.235359
	.218602	.202340	.186596	.171371	.156730	.142674	.129207	.116351
	.104130	.092568	.081668	.071455	.061912	.053105	.044940	.037523
	.030821	.024841	.019557	.014959	.011060	.007786	.005125	.002995
	.001399	.000360	.000000	.000360	.001399	.002995	.005125	.007786
	.011060	.014959	.019557	.024841	.030821	.037523	.044940	.053105
	.061912	.071455	.081668	.092568	.104130	.116351	.129207	.142674
	.156730	.171371	.186596	.202340	.218602	.235359	.252590	.270271
	.288381	.306878	.325740	.344946	.364455	.384243	.404292	.424539
	.444984	.465566	.406263	.507056	.527884	.540725	.569541	.590310

Figure A.1- Card input for sample case

Item No.								
7	.610951	.631465	.651791	.671909	.691779	.711341	.730593	.749457
	.767891	.785896	.803411	.820417	.836834	.852680	.867877	.882424
	.896242	.909350	.921710	.933260	.944001	.953914	.962959	.971116
	.978365	.984707	.990123	.994611	.998133			
8	.449164	.440979	.368724	.356677	.323122	.288384	.256968	.228251
	.201234	.174994	.152770	.131478	.107828	.088414	.069020	.049604
	.030206	.011790	-.005341	-.023660	-.041742	-.059474	-.074743	-.092628
	-.109767	-.125161	-.141841	-.156432	-.170621	-.181949	-.190490	-.196776
	-.198872	-.203294	-.235719	-.367492	-.586069	-.746810	-.804612	-.822557
	-.830010	-.830430	-.828029	-.823224	-.815167	-.805731	-.794610	-.780953
	-.765371	-.748500	-.728200	-.707375	-.685933	-.658961	-.629807	-.598306
	-.565587	-.528097	-.483403	-.441165	-.394459	-.336198	-.271604	-.211304
	-.136609	-.051358	.052134	.163559	.296591	.449001	.621890	.810551
	.994877	1.129638	1.169160	1.096662	.933160	.727769	.526205	.346353
	.190742	.056809	-.053975	-.156994	-.241464	-.316023	-.377321	-.443985
	-.500232	-.542507	-.588047	-.626762	-.662591	-.693996	-.723334	-.751171
	-.777519	-.798919	-.819680	-.839982	-.857221	-.873286	-.888300	-.901015
	-.912830	-.923922	-.932634	-.940026	-.946688	-.949963	-.952335	-.951561
	-.946329	-.937133	-.917745	-.885770	-.780090	-.450920	-.135657	-.055363
	-.049679	-.054497	-.059285	-.058713	-.056033	-.048960	-.039039	-.030023
	-.017360	-.003396	.011570	.025968	.042113	.059613	.077446	.095514
	.113310	.136492	.156974	.178507	.204143	.230640	.258908	.289933
	.324329	.357590	.369390	.449350	.449164			
6	.800000	.000000						
7	.999495	.997758	.995132	.991597	.987091	.981656	.975290	.968014
	.959827	.950749	.940799	.930018	.918425	.906020	.892863	.878994
	.864393	.849140	.833235	.816753	.799690	.782110	.764038	.745536
	.726603	.707279	.687646	.667702	.647510	.627108	.606518	.585801
	.564955	.544062	.523144	.502239	.481369	.460595	.439937	.419417
	.399095	.378972	.359110	.339530	.320253	.301322	.282758	.264582
	.246838	.229547	.212731	.196413	.180616	.165342	.150655	.136558
	.123054	.110166	.097919	.086338	.075427	.065211	.055678	.046895
	.038773	.031422	.024818	.018980	.013897	.009585	.006089	.003379
	.001483	.000362	.000000	.000362	.001483	.003379	.006089	.009586
	.013897	.018980	.024818	.031422	.038773	.046895	.055678	.065211
	.075427	.086338	.097919	.110166	.123054	.136558	.150655	.165342
	.180616	.196413	.212731	.229547	.246838	.264582	.282758	.301322
	.320253	.339530	.359110	.378972	.399095	.419417	.439937	.460595
	.481369	.502239	.523144	.544062	.564955	.585801	.606518	.627108
	.647510	.667702	.687646	.707279	.726603	.745536	.764038	.782110
	.799690	.816758	.833235	.849140	.864393	.878994	.892863	.906020
	.918425	.930018	.940799	.950749	.959827	.968014	.975290	.981656
	.987091	.991597	.995132	.997758	.999495			
8	.436539	.455539	.406037	.392728	.362350	.326615	.294714	.265263
	.238623	.213577	.189264	.164999	.144313	.124278	.101626	.083098
	.064554	.045974	.027430	.009934	-.006143	-.023238	-.039876	-.055862
	-.068951	-.084043	-.097616	-.108363	-.118872	-.125296	-.128789	-.127030
	-.123011	-.131088	-.199768	-.450296	-.729836	-.839648	-.868610	-.882942
	-.889736	-.891484	-.891050	-.886414	-.880317	-.872670	-.862403	-.851047
	-.838181	-.822989	-.805990	-.787772	-.766224	-.744070	-.721154	-.692722
	-.662052	-.629341	-.593876	-.553475	-.510373	-.459592	-.407829	-.341769
	-.267177	-.193761	-.101301	.008967	.147792	.309038	.505174	.724631
	.941542	1.107686	1.170402	1.107686	.941542	.724631	.505174	.309038
	.147792	.008967	-.101302	-.193761	-.267177	-.341769	-.407829	-.459592
	-.510373	-.553475	-.593876	-.629341	-.662053	-.692722	-.721154	-.744070
	-.766224	-.787772	-.805991	-.822989	-.838182	-.851047	-.862403	-.872670
	-.880317	-.886415	-.891050	-.891484	-.889736	-.882942	-.868610	-.839649
	-.729837	-.450298	-.199788	-.131088	-.123011	-.127030	-.128789	-.125296
	-.118372	-.108362	-.097616	-.084043	-.068951	-.055862	-.039376	-.023238
	-.006143	.009734	.027430	.045974	.064554	.083098	.101626	.124277

Figure A.1- Continued

Item No.								
8	.144313	.164999	.189264	.213576	.238623	.265263	.294714	.326615
	.362350	.392727	.406037	.486539	.486539			
6	.800000	.100000						
7	.998531	.996261	.992734	.988238	.982815	.976464	.969203	.961034
	.951976	.942049	.931291	.919724	.907346	.894219	.880381	.865812
	.850593	.834722	.818282	.801251	.783710	.765678	.747217	.728326
	.709045	.689455	.669555	.649407	.629051	.608507	.587835	.567035
	.546189	.525317	.504458	.483635	.462907	.442295	.421820	.401543
	.381464	.361646	.342110	.322876	.303986	.285462	.267326	.249620
	.232366	.215585	.199302	.183537	.168293	.153635	.139563	.126082
	.113216	.100986	.089419	.078517	.068305	.058770	.049977	.041834
	.034450	.027794	.021880	.016686	.012213	.008485	.005447	.003104
	.001403	.000361	.000000	.000361	.001403	.003104	.005447	.008485
	.012213	.016686	.021880	.027794	.034450	.041834	.049977	.058770
	.068305	.078517	.089419	.100986	.113216	.126082	.139563	.153635
	.168293	.183537	.199302	.215585	.232366	.249620	.267326	.285462
	.303986	.322876	.342110	.361646	.381464	.401543	.421820	.442295
	.462907	.483635	.504458	.525317	.546189	.567035	.587835	.608507
	.629051	.649407	.669555	.689455	.709045	.728326	.747217	.765678
	.783710	.801251	.818282	.834722	.850593	.865812	.880381	.894219
	.907346	.919724	.931291	.942049	.951976	.961034	.969203	.976464
	.982815	.988238	.992734	.996261	.998881			
8	.458593	.458791	.378081	.371267	.338852	.305476	.273840	.245302
	.218790	.193380	.168300	.146963	.126374	.103259	.084303	.065328
	.046302	.027281	.009255	-.007440	-.025268	-.042783	-.059836	-.074259
	-.091057	-.106811	-.120304	-.134618	-.145906	-.155618	-.161043	-.162445
	-.162500	-.160324	-.216525	-.409216	-.665613	-.793270	-.838444	-.852963
	-.858704	-.860935	-.858258	-.853677	-.847175	-.837775	-.827215	-.815116
	-.800639	-.784352	-.766860	-.746036	-.724693	-.702694	-.675207	-.645547
	-.614010	-.580003	-.541413	-.500299	-.451419	-.402259	-.340192	-.270822
	-.204426	-.121671	-.025336	.093513	.226491	.387549	.572665	.774910
	.971786	1.120326	1.170101	1.103351	.940608	.733454	.524063	.336348
	.173240	.040723	-.078005	-.173234	-.256245	-.322903	-.393605	-.455428
	-.501918	-.549224	-.589936	-.627654	-.660684	-.691319	-.720232	-.747349
	-.769284	-.790569	-.811391	-.829099	-.845728	-.860664	-.873407	-.884928
	-.895225	-.904302	-.911372	-.917277	-.919415	-.920039	-.916639	-.907461
	-.892103	-.860794	-.745397	-.440787	-.163682	-.093649	-.088298	-.093776
	-.096178	-.095105	-.088954	-.081614	-.070719	-.057721	-.046263	-.031594
	-.016004	.000265	.015695	.032676	.050827	.069131	.087505	.105956
	.123637	.142566	.169196	.194712	.219847	.246217	.274591	.306084
	.337369	.371654	.378391	.456995	.456893			
6	.800000	.300000						
7	.999495	.997755	.995132	.991597	.987091	.981656	.975290	.968014
	.959827	.950749	.940799	.930018	.918425	.906020	.892663	.878994
	.864393	.849140	.833235	.816758	.799690	.782110	.764039	.745536
	.726603	.707279	.687646	.667702	.647510	.627108	.606518	.585801
	.564955	.544062	.523144	.502239	.481369	.460595	.439937	.419417
	.399095	.378972	.359110	.339530	.320253	.301322	.282758	.264582
	.246833	.229547	.212731	.196413	.180616	.165342	.150655	.136558
	.123054	.110166	.097919	.086330	.075427	.065211	.055678	.046895
	.038773	.031422	.024818	.018980	.013897	.009586	.006059	.003379
	.001483	.000362	.000000	.000362	.001483	.003379	.006059	.009586
	.013897	.018980	.024818	.031422	.038773	.046895	.055678	.065211
	.075427	.086330	.097919	.110166	.123054	.136558	.150655	.165342
	.180616	.196413	.212731	.229547	.246833	.264582	.282758	.301322
	.320253	.339530	.359110	.378972	.399095	.419417	.439937	.460595
	.481369	.502239	.523144	.544062	.564955	.585801	.606518	.627108
	.647510	.667702	.687646	.707279	.726603	.745536	.764039	.782110
	.799690	.816758	.833235	.849140	.864393	.878994	.892663	.906020
	.918425	.930018	.940799	.950749	.959827	.968014	.975290	.981656

Figure A.1- Continued

Item No.								
7	.987091	.991597	.995132	.997758	.999495			
8	.489105	.488856	.402558	.395215	.364845	.329132	.297230	.267752
	.241058	.215931	.191510	.167103	.146225	.125956	.103014	.08[illegible]
	.065123	.045956	.026752	.008404	-.008728	-.027140	-.045429	-.063491
	-.079201	-.097655	-.115657	-.132155	-.150327	-.166937	-.183882	-.198952
	-.212493	-.225064	-.233421	-.233540	-.247696	-.284055	-.404125	-.589334
	-.720706	-.772628	-.793783	-.795047	-.794404	-.790553	-.782963	-.773702
	-.762569	-.748667	-.732627	-.715995	-.693040	-.671557	-.649105	-.620354
	-.585698	-.554654	-.516249	-.477942	-.432722	-.376478	-.326477	-.262355
	-.188043	-.114700	-.021243	.089693	.227969	.357372	.573277	.787676
	.928193	1.131475	1.167507	1.078406	.890077	.657697	.429117	.222601
	.066214	-.072675	-.182169	-.274184	-.348[illegible]5	-.4247[illegible]	-.488345	-.535603
	-.585009	-.627328	-.666135	-.699432	-.731236	-.760614	-.788219	-.810703
	-.832370	-.853357	-.871153	-.888087	-.903069	-.917736	-.930370	-.942268
	-.951914	-.960593	-.968506	-.973536	-.978179	-.980470	-.979557	-.976684
	-.966756	-.949158	-.918589	-.848190	-.600836	-.220297	-.035559	-.008065
	-.014402	-.021252	-.025646	-.024423	-.019218	-.013954	-.004353	.007052
	.019861	.032428	.046981	.063042	.079533	.096314	.113339	.134662
	.153582	.173302	.196701	.220255	.244641	.270691	.299605	.331015
	.366298	.396308	.409349	.489330	.489108			
6	.800000	.400000						
7	.998831	.996261	.992734	.988238	.982815	.976464	.969203	.961034
	.951976	.942049	.931291	.919724	.907346	.894219	.880381	.865812
	.850593	.834722	.818282	.801251	.783710	.765678	.747217	.728326
	.709045	.689455	.669555	.649407	.629051	.608507	.587835	.567035
	.546189	.525317	.504458	.483635	.462907	.442295	.421830	.401543
	.381464	.361646	.342110	.322876	.303936	.285462	.267326	.249620
	.232366	.215585	.199302	.183537	.168293	.153635	.139563	.126082
	.113216	.100986	.089419	.078517	.068305	.058770	.049977	.041934
	.034450	.027794	.021880	.016686	.012213	.008485	.005447	.003104
	.001403	.000361	.000000	.000361	.001403	.003104	.005447	.008485
	.012213	.016686	.021880	.027794	.034450	.041034	.049977	.058770
	.068305	.078517	.089419	.100986	.113216	.126082	.139563	.153635
	.168293	.183537	.199302	.215585	.232366	.249620	.267326	.285462
	.303906	.322876	.342110	.361646	.381464	.401543	.421830	.442295
	.462907	.483635	.504458	.525317	.546189	.567035	.587835	.608507
	.629051	.649407	.669555	.689455	.709045	.728326	.747217	.765678
	.783710	.801251	.818282	.834722	.850593	.865812	.880381	.894219
	.907346	.919724	.931291	.942049	.951976	.961034	.969203	.976464
	.982815	.988238	.992734	.996261	.998831			
8	.463906	.463542	.383210	.376335	.344041	.310753	.279223	.250763
	.224307	.198933	.173673	.152520	.131801	.108699	.089614	.070459
	.051192	.031852	.013404	-.003544	-.022303	-.040209	-.059045	-.075003
	-.093807	-.112203	-.129293	-.146268	-.166067	-.184700	-.202242	-.219409
	-.237169	-.252367	-.264630	-.275050	-.267069	-.310622	-.384371	-.520500
	-.651464	-.723049	-.749049	-.757014	-.757358	-.752204	-.744825	-.735153
	-.722462	-.707479	-.699930	-.670487	-.649353	-.627514	-.599640	-.560836
	-.535928	-.501070	-.461913	-.418310	-.367155	-.318745	-.258751	-.190464
	-.123351	-.039674	.057341	.175989	.308200	.466303	.645304	.836752
	1.016671	1.141602	1.165213	1.072227	.857264	.664673	.446882	.253724
	.059560	-.043323	-.161762	-.257318	-.340367	-.405038	-.476755	-.534029
	-.576506	-.625309	-.664559	-.700900	-.732686	-.762251	-.790133	-.816546
	-.838046	-.850656	-.878623	-.896170	-.913119	-.928684	-.942203	-.954687
	-.966554	-.976330	-.985336	-.993910	-.999744	-1.005513	-1.009427	-1.010893
	-1.011613	-1.007216	-.990251	-.981310	-.951180	-.898942	-.675924	-.267956
	-.014439	.020454	.026025	.017221	.012651	.013018	.014088	.020132
	.028577	.035929	.049469	.062311	.076941	.092261	.108999	.124370
	.145106	.163625	.183292	.206792	.230023	.256119	.283563	.314198
	.346700	.378313	.384611	.464269	.463906			
6	.800000	.600000						

Figure A.1- Continued

Item No.								
7	.998133	.994611	.990123	.984707	.978365	.971116	.962959	.953914
	.944001	.933260	.921710	.909350	.896242	.882424	.867877	.852680
	.836834	.820417	.803411	.785896	.767891	.749457	.730593	.711341
	.691779	.671909	.651791	.631465	.610951	.590310	.569541	.548725
	.527884	.507056	.486263	.465566	.444984	.424539	.404292	.384243
	.364455	.344946	.325740	.306878	.288381	.270271	.252590	.235359
	.218602	.202340	.186596	.171371	.156730	.142674	.129207	.116351
	.104130	.092568	.081668	.071455	.061912	.053105	.044940	.037523
	.030321	.024841	.019557	.014959	.011060	.007786	.005125	.002995
	.001399	.000360	.000000	.000360	.001399	.002995	.005125	.007786
	.011060	.014959	.019557	.024841	.030321	.037523	.044940	.053105
	.061912	.071455	.081668	.092568	.104130	.116351	.129207	.142674
	.156730	.171371	.186596	.202340	.218602	.235359	.252590	.270271
	.288381	.306878	.325740	.344946	.364455	.384243	.404292	.424539
	.444984	.465566	.486263	.507056	.527884	.548725	.569541	.590310
	.610951	.631465	.651791	.671909	.691779	.711341	.730593	.749457
	.767891	.785896	.803411	.820417	.836834	.852680	.867877	.882424
	.896242	.909350	.921710	.933260	.944001	.953914	.962959	.971116
	.978365	.984707	.990123	.994611	.998133			
8	.462483	.461570	.382264	.370167	.336918	.302577	.271523	.243132
	.216423	.190478	.168483	.147409	.124007	.104765	.085520	.066233
	.046939	.028541	.011355	-.007085	-.025401	-.043535	-.059445	-.078202
	-.096599	-.113795	-.133011	-.151223	-.170603	-.189406	-.208636	-.229710
	-.250142	-.270238	-.291004	-.312540	-.331581	-.351860	-.373582	-.403607
	-.454352	-.519560	-.581649	-.625410	-.647867	-.656544	-.656523	-.649851
	-.638908	-.625234	-.606912	-.587538	-.567393	-.540859	-.511204	-.479994
	-.446358	-.407620	-.365183	-.319089	-.274597	-.218665	-.155032	-.093624
	-.017795	.068219	.171463	.282458	.413040	.560212	.723428	.895624
	1.054411	1.155458	1.157109	1.047605	.852882	.6[illegible]219	.410777	.224683
	.066900	-.066937	-.176675	-.280304	-.366630	-.442649	-.499694	-.553762
	-.611902	-.653193	-.695634	-.731009	-.764265	-.793388	-.820540	-.846013
	-.870094	-.889922	-.909713	-.929431	-.946512	-.962757	-.977730	-.990991
	-1.003343	-1.015392	-1.025594	-1.035091	-1.044424	-1.051292	-1.058472	-1.064373
	-1.068684	-1.073640	-1.075761	-1.077063	-1.076656	-1.073327	-1.063527	-1.045402
	-1.011530	-.950877	-.606516	-.212324	.066583	.111356	.110028	.101566
	.097637	.097297	.100041	.104126	.111381	.121190	.132408	.144752
	.158122	.176327	.192981	.211133	.233658	.257373	.283129	.311842
	.344092	.375503	.385985	.463397	.462483			
6	.800000	.700000						
7	.997307	.992822	.987410	.981072	.973828	.965677	.956638	.946732
	.935998	.924455	.912104	.899005	.885196	.870659	.855472	.839636
	.823231	.806236	.788732	.770739	.752317	.733466	.714226	.694678
	.674821	.654716	.634403	.613903	.593275	.572520	.551718	.530890
	.510077	.489297	.468614	.448046	.427614	.407381	.387345	.367569
	.348073	.328880	.310029	.291544	.273445	.255774	.238553	.221805
	.205551	.189815	.174597	.159960	.145909	.132444	.119590	.107368
	.095802	.084896	.074673	.065117	.056291	.048101	.040650	.033903
	.027865	.022504	.017806	.013774	.010327	.007438	.005001	.002992
	.001397	.000359	.000000	.000359	.001397	.002992	.005001	.007438
	.010327	.013774	.017806	.022504	.027865	.033903	.040650	.048101
	.056291	.065117	.074673	.084896	.095802	.107368	.119590	.132444
	.145909	.159960	.174597	.189815	.205551	.221805	.238553	.255774
	.273445	.291544	.310029	.328880	[illegible]8073	.367569	.387345	.407381
	.427614	.448046	.468614	.489297	.510077	.530890	.551718	.572520
	.593275	.613903	.634403	.654716	.674821	.694678	.714226	.733466
	.752317	.770739	.788732	.806236	.823231	.839636	.855472	.870659
	.885196	.899005	.912104	.924455	.935998	.946732	.956638	.965677
	.973828	.981072	.987410	.992822	.997307			
8	.668728	.668307	.5223[illegible]	[illegible]943	.369762	.314595	.273042	.239321
	.209654	.185547	.163[illegible]	[illegible]39135	.119414	.099830	.080298	.062812

Figure A.1- Continued

8	.042266	.024934	.006399	-.011958	-.030082	-.045955	-.064578	-.082762
	-.099687	-.118538	-.136324	-.155210	-.173483	-.192160	-.212669	-.232627
	-.252465	-.273361	-.295668	-.316378	-.338889	-.361301	-.383949	-.409232
	-.437231	-.471741	-.508264	-.539433	-.562879	-.577161	-.581808	-.579150
	-.571404	-.557257	-.540964	-.523283	-.498949	-.471497	-.441655	-.409009
	-.372779	-.334533	-.290600	-.249155	-.196606	-.137268	-.081064	-.012045
	.064911	.156050	.250797	.360544	.482506	.618924	.770906	.932586
	1.076892	1.162004	1.149382	1.027122	.829499	.598496	.390563	.198798
	.049066	-.077266	-.167047	-.280101	-.373953	-.451564	-.515760	-.564158
	-.619415	-.668576	-.705960	-.744879	-.777275	-.808043	-.834895	-.859770
	-.883137	-.906457	-.925930	-.944910	-.963698	-.979917	-.995537	-1.010093
	-1.023392	-1.035272	-1.047226	-1.057403	-1.066934	-1.076385	-1.083463	-1.090957
	-1.097319	-1.102275	-1.108152	-1.111634	-1.114988	-1.117839	-1.119957	-1.119530
	-1.117793	- .112354	-1.098409	-1.070169	-1.009848	-.890889	-.417645	.062612
	.154654	.163569	.160082	.157352	.156120	.158521	.164013	.171487
	.180676	.191367	.207425	.222816	.240828	.265001	.293844	.331050
	.382321	.450996	.528020	.669150	.668728			

Figure A.1- Concluded

```
**************************************************
*                                                *
*                PROGRAM PERTURB                 *
*                                                *
*     CALCULATES NONLINEAR SINGLE-PARAMETER      *
*                                                *
*       CONTINUOUS OR DISCONTINUOUS              *
*                                                *
*          PERTURBATION SOLUTIONS                *
*                                                *
*     WHICH REPRESENT A CHANGE IN EITHER         *
*                                                *
*       GEOMETRY OR FLOW CONDITIONS              *
*                                                *
*  BY EMPLOYING A STRAINED-COORDINATE PROCEDURE  *
*                                                *
*  UTILIZING A UNIT PERTURBATION DETERMINED FROM *
*                                                *
*         TWO PREVIOUSLY CALCULATED              *
*                                                *
*     'BASE' AND 'CALIBRATION' SOLUTIONS         *
*                                                *
*  DISPLACED FROM ONE ANOTHER BY SOME REASONABLE *
*                                                *
*     CHANGE IN GEOMETRY OR FLOW CONDITION       *
*                                                *
*                                                *
*                WRITTEN BY                      *
*                                                *
*    JAMES P. ELLIOTT AND STEPHEN S. STAHARA     *
*                                                *
*                                                *
*    NIELSEN ENGINEERING AND RESEARCH, INC.      *
*                                                *
*        MOUNTAIN VIEW, CALIFORNIA               *
*                                                *
**************************************************

*****************************************************************************************************
*                     SAMPLE CASE - ALPHA PERTURBATION FOR MULTI-SHOCK ISOLATED AIRFOIL FLOW        *
*****************************************************************************************************

<<<<<<<<<< LIST OF INPUT PARAMETERS >>>>>>>>>>

    N = 149

    A = -1.0    B =  1.0

    BASE SOLUTION:        M0 = 0.8000     Q0 = 0.5000

    CALIBRATION SOLN:     M1 = 0.8000     Q1 = 0.2000
```

Figure A.2- Abbreviated print output for sample case

```
<<<<<<<<<<< CRITICAL VALUES OF CP >>>>>>>>>>

     BASE SOLUTION:         CPCRIT = -0.4346

     CALIBRATION SOLN:      CPCRIT = -0.4346


<<<<< LOCATIONS OF MIN., MAX., AND CRITICAL PTS. >>>>>
  (* DENOTES POINT ON LOWER SURFACE)

     BASE SOLUTION:

          MINIMUM AT X = 0.4606     (POINT #112)
          MAXIMUM AT X = 0.0000     (POINT # 75)
          2 CRITICAL POINT(S):
              1ST AT X = 0.3924*    (AFTER POINT # 41)
              2ND AT X = 0.6288     (AFTER POINT #120)

     CALIBRATION SOLN:

          MINIMUM AT X = 0.4043     (POINT #111)
          MAXIMUM AT X = 0.0000     (POINT # 75)
          2 CRITICAL POINT(S):
              1ST AT X = 0.4592*    (AFTER POINT # 36)
              2ND AT X = 0.5498     (AFTER POINT #118)


<<<<<<<<<< STRAINING POINTS SELECTED >>>>>>>>>>

     NUMBER OF FIXED POINTS : 5

     FIXED POINTS SELECTED (IN ADDITION TO END POINTS) :

          POINT OF MAXIMUM CP
          CPCRIT (1ST OF 2)
          CPCRIT (2ND OF 2)


<<<<<<<<<< LOCATION OF FIXED POINTS >>>>>>>>>>


  (* DENOTES POINT ON LOWER SURFACE)

     BASE SOLUTION:

          XFIX(1) = 1.0000*
          XFIX(2) = 0.3924*
          XFIX(3) = 0.0000
          XFIX(4) = 0.6288
          XFIX(5) = 1.0000

     CALIBRATION SOLN:

          XFIX(1) = 1.0000*
          XFIX(2) = 0.4592*
          XFIX(3) = 0.0000
          XFIX(4) = 0.5498
          XFIX(5) = 1.0000
```

Figure A.2- Continued

```
***************************
* UNIT PERTURBATION OF CP *
*            &            *
* UNIT STRAINING OF XBASE *
***************************
```

POINT	XBASE	XSTRUNIT	CPUNIT
1	0.9995	0.9996	0.1516
2	0.9978	0.9980	0.1503
3	0.9951	0.9957	-0.1153
4	0.9916	0.9925	-0.0350
5	0.9871	0.9885	0.0192
6	0.9817	0.9837	-0.0527
7	0.9733	0.9780	-0.0591
8	0.9680	0.9715	-0.0533
9	0.9[illegible]98	0.9642	-0.0470
10	0.9507	0.9562	-0.0418
11	0.9408	0.9473	-0.0390
12	0.9300	0.9377	-0.0386
13	0.9184	0.9274	-0.0339
14	0.9060	0.9164	-0.0339
15	0.8929	0.9046	-0.0415
16	0.8790	0.8923	-0.0351
17	0.8644	0.8793	-0.0381
18	0.8491	0.8657	-0.0417
19	0.8332	0.8516	-0.0455
20	0.8168	0.8369	-0.0468
21	0.7997	0.8217	-0.0471
22	0.7821	0.8061	-0.0556
23	0.7640	0.7900	-0.0611
24	0.7455	0.7735	-0.0662
25	0.7266	0.7567	-0.0647
26	0.7073	0.7395	-0.0775
27	0.6876	0.7220	-0.0888
28	0.6677	0.7043	-0.0928
29	0.6475	0.6863	-0.1067
30	0.6271	0.6681	-0.1201
31	0.6065	0.6498	-0.1349
32	0.5858	0.6314	-0.1532
33	0.5650	0.6128	-0.1734
34	0.5441	0.5942	-0.2064
35	0.5231	0.5756	-0.2428
36	0.5022	0.5570	-0.2809
37	0.4814	0.5384	-0.3257
38	0.4606	0.5199	-0.3711
39	0.4399	0.5015	-0.3818
40	0.4194	0.4833	-0.3191
41	0.3991	0.4652	-0.1193
42	0.3790	0.4435	0.2948
43	0.3591	0.4203	0.5270
44	0.3395	0.3974	0.4753
45	0.3203	0.3748	0.4194
46	0.3013	0.3526	0.3944
47	0.2828	0.3309	0.3886
48	0.2646	0.3096	0.3889
49	0.2468	0.2889	0.3877
50	0.2295	0.2686	0.3917
51	0.2127	0.2490	0.3971

Figure A.2- Continued

52	0.1964	0.2299	0.3990
53	0.1806	0.2114	0.4079
54	0.1653	0.1935	0.4105
55	0.1507	0.1763	0.4084
56	0.1366	0.1598	0.4252
57	0.1231	0.1440	0.4357
58	0.1102	0.1289	0.4411
59	0.0979	0.1146	0.4454
60	0.0863	0.1010	0.4572
61	0.0754	0.0883	0.4672
62	0.0652	0.0763	0.4799
63	0.0557	0.0652	0.4718
64	0.0469	0.0549	0.4757
65	0.0388	0.0454	0.4779
66	0.0314	0.0368	0.4874
67	0.0248	0.0290	0.4954
68	0.0190	0.0222	0.4945
69	0.0139	0.0163	0.5134
70	0.0096	0.0112	0.5078
71	0.0061	0.0071	0.4605
72	0.0034	0.0040	0.3529
73	0.0015	0.0017	0.2112
74	0.0004	0.0004	0.0803
75	0.0000	0.0000	-0.0247
76	0.0004	0.0003	-0.1734
77	0.0015	0.0013	-0.3329
78	0.0034	0.0030	-0.4227
79	0.0061	0.0053	-0.4710
80	0.0096	0.0084	-0.4994
81	0.0139	0.0121	-0.4931
82	0.0190	0.0166	-0.5033
83	0.0248	0.0217	-0.4911
84	0.0314	0.0275	-0.4720
85	0.0388	0.0339	-0.4489
86	0.0469	0.0410	-0.4620
87	0.0557	0.0487	-0.4517
88	0.0652	0.0570	-0.4046
89	0.0754	0.0659	-0.4024
90	0.0863	0.0755	-0.3963
91	0.0979	0.0856	-0.3821
92	0.1102	0.0963	-0.3693
93	0.1231	0.1076	-0.3594
94	0.1366	0.1194	-0.3552
95	0.1507	0.1317	-0.3520
96	0.1653	0.1446	-0.3368
97	0.1806	0.1579	-0.3235
98	0.1964	0.1717	-0.3246
99	0.2127	0.1860	-0.3201
100	0.2295	0.2007	-0.3143
101	0.2468	0.2158	-0.3123
102	0.2646	0.2313	-0.3079
103	0.2828	0.2472	-0.3037
104	0.3013	0.2634	-0.3028
105	0.3203	0.2800	-0.2994
106	0.3395	0.2969	-0.2961
107	0.3591	0.3140	-0.2961
108	0.3790	0.3313	-0.2930
109	0.3991	0.3489	-0.2937
110	0.4194	0.3667	-0.2927
111	0.4399	0.3846	-0.2942

Figure A.2- Continued

112	0.4606	0.4027	-0.2989
113	0.4314	0.4209	-0.3000
114	0.5022	0.4391	-0.3053
115	0.5231	0.4574	-0.3079
116	0.5441	0.4757	-0.3077
117	0.5650	0.4939	-0.2921
118	0.5858	0.5122	-0.2933
119	0.6065	0.5303	-0.3833
120	0.6271	0.5483	-0.0654
121	0.6475	0.5724	0.2949
122	0.6677	0.5969	0.3781
123	0.6876	0.6211	0.3958
124	0.7073	0.6449	0.3910
125	0.7266	0.6684	0.3758
126	0.7455	0.6913	0.3521
127	0.7640	0.7138	0.3283
128	0.7821	0.7357	0.3056
129	0.7997	0.7570	0.2892
130	0.8168	0.7777	0.2640
131	0.8332	0.7977	0.2435
132	0.8491	0.8170	0.2301
133	0.8644	0.8355	0.2154
134	0.8790	0.8532	0.1976
135	0.8929	0.8700	0.1802
136	0.9060	0.8860	0.1773
137	0.9184	0.9011	0.1615
138	0.9300	0.9151	0.1423
139	0.9408	0.9282	0.1308
140	0.9507	0.9403	0.1259
141	0.9598	0.9513	0.1090
142	0.9680	0.9612	0.0929
143	0.9753	0.9700	0.0783
144	0.9817	0.9777	0.0614
145	0.9871	0.9843	0.0620
146	0.9916	0.9898	0.1164
147	0.9951	0.9941	-0.0799
148	0.9978	0.9973	0.1531
149	0.9995	0.9994	0.1520

Figure A.2- Continued

```
***************************
* OUTPUT FOR CASE #1 OF 6 *
***************************

     M2 = 0.6000

     Q2 = 0.0000

     CPCRIT = -0.4346


<<<<< LOCATIONS OF MIN., MAX., AND CRITICAL PTS. >>>>>
  (* DENOTES POINT ON LOWER SURFACE)

     PERTURB'TION SOLN:

          MINIMUM AT X = 0.4112*    (POINT # 45)
          MAXIMUM AT X = 0.0000*    (POINT # 75)
          2 CRITICAL POINT(S):
               1ST AT X = 0.5024*    (AFTER POINT # 41)
               2ND AT X = 0.4961     (AFTER POINT #120)

     COMPARISON SOLN:

          MINIMUM AT X = 0.3790*    (POINT # 42)
          MAXIMUM AT X = 0.0000     (POINT # 75)
          2 CRITICAL POINT(S):
               1ST AT X = 0.5035*    (AFTER POINT # 35)
               2ND AT X = 0.5035     (AFTER POINT #114)
```

POINT	XBASE	CPBASE	XCALB	CPCALB	XPERT	CPPERT	XCHEK	CPCHEK	CPPERT(INT)
1	0.9995	0.4947	0.9981	0.4492	0.9996	0.4189	0.9995	0.4865	0.4189
2	0.9978	0.4943	0.9946	0.4490	0.9982	0.4191	0.9978	0.4865	0.4293
3	0.9951	0.4144	0.9901	0.3687	0.9960	0.4721	0.9951	0.4060	0.4556
4	0.9916	0.4011	0.9847	0.3567	0.9931	0.4186	0.9916	0.3927	0.3946
5	0.9871	0.3709	0.9784	0.3231	0.9895	0.3613	0.9871	0.3623	0.3615
6	0.9817	0.3354	0.9711	0.2894	0.9850	0.3618	0.9817	0.3266	0.3433
7	0.9753	0.3037	0.9630	0.2570	0.9798	0.3332	0.9753	0.2947	0.3087
8	0.9680	0.2744	0.9539	0.2283	0.9739	0.3010	0.9680	0.2653	0.2750
9	0.9598	0.2478	0.9440	0.2012	0.9672	0.2713	0.9598	0.2306	0.2440
10	0.9507	0.2228	0.9333	0.1750	0.9598	0.2438	0.9507	0.2136	0.2155
11	0.9408	0.1986	0.9217	0.1528	0.9517	0.2180	0.9408	0.1893	0.1886
12	0.9300	0.1743	0.9093	0.1315	0.9428	0.1936	0.9300	0.1650	0.1638
13	0.9184	0.1535	0.8962	0.1078	0.9334	0.1705	0.9184	0.1443	0.1418
14	0.9060	0.1334	0.8824	0.0884	0.9232	0.1503	0.9060	0.1243	0.1187
15	0.8929	0.1106	0.8679	0.0690	0.9125	0.1313	0.8929	0.1016	0.0972
16	0.8790	0.0918	0.8527	0.0496	0.9012	0.1093	0.8790	0.0831	0.0777
17	0.8644	0.0728	0.8368	0.0302	0.8893	0.0919	0.8644	0.0646	0.0581
18	0.8491	0.0538	0.8204	0.0118	0.8768	0.0746	0.8491	0.0460	0.0382
19	0.8332	0.0346	0.8034	-0.0053	0.8638	0.0574	0.8332	0.0274	0.0195
20	0.8168	0.0163	0.7859	-0.0237	0.8504	0.0397	0.8168	0.0099	0.0029
21	0.7997	-0.0009	0.7679	-0.0417	0.8364	0.0227	0.7997	-0.0061	-0.0151
22	0.7821	-0.0193	0.7495	-0.0595	0.8221	0.0005	0.7821	-0.0232	-0.0338
23	0.7640	-0.0377	0.7306	-0.0747	0.8073	-0.0072	0.7640	-0.0399	-0.0496
24	0.7455	-0.0559	0.7113	-0.0926	0.7922	-0.0228	0.7455	-0.0559	-0.0645
25	0.7266	-0.0719	0.6918	-0.1098	0.7767	-0.0396	0.7266	-0.0690	-0.0818

Figure A.2- Continued

26	0.7073	-0.0908	0.6719	-0.1252	0.7609	-0.0520	0.7073	-0.0840	-0.0961
27	0.6876	-0.1094	0.6518	-0.1418	0.7449	-0.0650	0.6876	-0.0976	-0.1099
28	0.6677	-0.1267	0.6315	-0.1564	0.7286	-0.0803	0.6677	-0.1084	-0.1225
29	0.6475	-0.1460	0.6110	-0.1706	0.7121	-0.0927	0.6475	-0.1189	-0.1332
30	0.6271	-0.1643	0.5903	-0.1819	0.6955	-0.1043	0.6271	-0.1253	-0.1305
31	0.6065	-0.1838	0.5695	-0.1905	0.6787	-0.1163	0.6065	-0.1288	-0.1391
32	0.5858	-0.2024	0.5487	-0.1968	0.6617	-0.1259	0.5858	-0.1270	-0.1356
33	0.5650	-0.2213	0.5279	-0.1989	0.6447	-0.1347	0.5650	-0.1230	-0.1284
34	0.5441	-0.2417	0.5071	-0.2033	0.6277	-0.1385	0.5441	-0.1311	-0.1347
35	0.5231	-0.2608	0.4863	-0.2357	0.6106	-0.1394	0.5231	-0.1998	-0.2159
36	0.5022	-0.2786	0.4656	-0.3675	0.5935	-0.1381	0.5022	-0.4503	-0.4368
37	0.4814	-0.2955	0.4450	-0.5861	0.5765	-0.1327	0.4814	-0.7298	-0.6696
38	0.4606	-0.3119	0.4245	-0.7468	0.5595	-0.1263	0.4606	-0.8396	-0.8651
39	0.4399	-0.3264	0.4043	-0.8046	0.5426	-0.1355	0.4399	-0.8686	-0.8992
40	0.4194	-0.3504	0.3842	-0.8226	0.5259	-0.1909	0.4194	-0.8829	-0.9086
41	0.3991	-0.4033	0.3645	-0.8300	0.5093	-0.3436	0.3991	-0.8897	-0.9096
42	0.3790	-0.4976	0.3449	-0.8304	0.4866	-0.6450	0.3790	-0.8915	-0.9082
43	0.3591	-0.6009	0.3257	-0.8280	0.4611	-0.8644	0.3591	-0.8910	-0.9056
44	0.3395	-0.6682	0.3069	-0.8232	0.4359	-0.9059	0.3395	-0.8064	-0.9016
45	0.3203	-0.7003	0.2884	-0.8152	0.4112	-0.9100	0.3203	-0.8803	-0.8942
46	0.3013	-0.7119	0.2703	-0.8057	0.3869	-0.9091	0.3013	-0.8727	-0.8860
47	0.2828	-0.7121	0.2526	-0.7946	0.3630	-0.9064	0.2828	-0.8624	-0.8765
48	0.2646	-0.7073	0.2354	-0.7810	0.3397	-0.9017	0.2646	-0.8510	-0.8648
49	0.2468	-0.6991	0.2186	-0.7654	0.3169	-0.8929	0.2468	-0.8382	-0.8514
50	0.2295	-0.6872	0.2023	-0.7485	0.2947	-0.8830	0.2295	-0.8230	-0.8370
51	0.2127	-0.6726	0.1866	-0.7282	0.2731	-0.8711	0.2127	-0.8060	-0.8197
52	0.1964	-0.6561	0.1714	-0.7074	0.2522	-0.8556	0.1964	-0.7878	-0.7995
53	0.1806	-0.6355	0.1567	-0.6859	0.2319	-0.8395	0.1806	-0.7662	-0.7814
54	0.1653	-0.6140	0.1427	-0.6590	0.2123	-0.8192	0.1653	-0.7441	-0.7601
55	0.1507	-0.5916	0.1292	-0.6298	0.1934	-0.7958	0.1507	-0.7212	-0.7348
56	0.1366	-0.5629	0.1164	-0.5989	0.1753	-0.7755	0.1366	-0.6927	-0.7068
57	0.1231	-0.5308	0.1041	-0.5656	0.1580	-0.7487	0.1231	-0.6621	-0.6767
58	0.1102	-0.4968	0.0926	-0.5280	0.1414	-0.7174	0.1102	-0.6293	-0.6450
59	0.0979	-0.4605	0.0817	-0.4884	0.1257	-0.6832	0.0979	-0.5939	-0.6090
60	0.0863	-0.4184	0.0715	-0.4412	0.1108	-0.6470	0.0863	-0.5535	-0.5689
61	0.0754	-0.3723	0.0619	-0.3945	0.0968	-0.6058	0.0754	-0.5104	-0.5224
62	0.0652	-0.3197	0.0531	-0.3362	0.0837	-0.5597	0.0652	-0.4596	-0.4704
63	0.0557	-0.2688	0.0449	-0.2716	0.0715	-0.5047	0.0557	-0.4078	-0.4116
64	0.0469	-0.2053	0.0375	-0.2113	0.0602	-0.4431	0.0469	-0.3418	-0.3492
65	0.0388	-0.1317	0.0308	-0.1366	0.0498	-0.3706	0.0388	-0.2672	-0.2838
66	0.0314	-0.0567	0.0248	-0.0514	0.0403	-0.3004	0.0314	-0.1938	-0.2038
67	0.0248	0.0373	0.0196	0.0521	0.0319	-0.2104	0.0248	-0.1013	-0.1055
68	0.0190	0.1485	0.0150	0.1636	0.0244	-0.0988	0.0190	0.0090	0.0069
69	0.0139	0.2859	0.0111	0.2966	0.0178	0.0292	0.0139	0.1478	0.1436
70	0.0096	0.4435	0.0078	0.4490	0.0123	0.1896	0.0096	0.3090	0.3170
71	0.0061	0.6300	0.0051	0.6219	0.0078	0.3998	0.0061	0.5052	0.5266
72	0.0034	0.8314	0.0030	0.8106	0.0043	0.6549	0.0034	0.7246	0.7570
73	0.0015	1.0193	0.0014	0.9949	0.0019	0.9137	0.0015	0.9415	0.9699
74	0.0004	1.1454	0.0004	1.1296	0.0005	1.1052	0.0004	1.1077	1.1206
75	0.0000	1.1617	0.0000	1.1692	0.0000	1.1740	0.0000	1.1704	1.1739
76	0.0004	1.0535	0.0004	1.0967	0.0003	1.1402	0.0004	1.1077	1.1294
77	0.0015	0.8493	0.0014	0.9332	0.0012	1.[illegible]158	0.0015	0.9415	0.9745
78	0.0034	0.6062	0.0030	0.7278	0.0027	0.8175	0.0034	0.7246	0.7479
79	0.0061	0.3715	0.0051	0.5262	0.0048	0.6070	0.0061	0.5052	0.5197
80	0.0096	0.1683	0.0078	0.3464	0.0076	0.4180	0.0096	0.3090	0.3201
81	0.0139	0.0054	0.0111	0.1907	0.0110	0.2519	0.0139	0.1478	0.1549
82	0.0190	-0.1336	0.0150	0.0568	0.0150	0.1181	0.0190	0.0090	0.0185
83	0.0248	-0.2430	0.0196	-0.0540	0.0196	0.0025	0.0248	-0.1013	-0.0993
84	0.0314	-0.3358	0.0248	-0.1570	0.0248	-0.0998	0.0314	-0.1938	-0.1939
85	0.0388	-0.4104	0.0308	-0.2415	0.0306	-0.1859	0.0388	-0.2672	-0.2684

Figure A.2-Continued

86	0.0469	-0.4834	0.0375	-0.3160	0.0371	-0.2524	0.0469	-0.3418	-0.7445
87	0.0557	-0.5434	0.0449	-0.3774	0.0440	-0.3175	0.0557	-0.4078	-0.4134
88	0.0652	-0.5903	0.0531	-0.4440	0.0515	-0.3800	0.0652	-0.4596	-0.4659
89	0.0754	-0.6388	0.0619	-0.5002	0.0596	-0.4376	0.0754	-0.5104	-0.5160
90	0.0863	-0.6794	0.0715	-0.5425	0.0682	-0.4812	0.0863	-0.5535	-0.5615
91	0.0979	-0.7167	0.0817	-0.5880	0.0774	-0.5256	0.0979	-0.5939	-0.6014
92	0.1102	-0.7492	0.0926	-0.6268	0.0871	-0.5645	0.1102	-0.6293	-0.6355
93	0.1231	-0.7793	0.1041	-0.6626	0.0973	-0.5996	0.1231	-0.6621	-0.6681
94	0.1366	-0.8075	0.1164	-0.6940	0.1080	-0.6299	0.1366	-0.6927	-0.7005
95	0.1507	-0.8341	0.1292	-0.7233	0.1191	-0.6581	0.1507	-0.7212	-0.7271
96	0.1653	-0.8557	0.1427	-0.7512	0.1307	-0.6874	0.1653	-0.7441	-0.7504
97	0.1806	-0.8763	0.1567	-0.7775	0.1428	-0.7145	0.1806	-0.7662	-0.7737
98	0.1964	-0.8968	0.1714	-0.7989	0.1553	-0.7345	0.1964	-0.7878	-0.7933
99	0.2127	-0.9149	0.1866	-0.8197	0.1682	-0.7548	0.2127	-0.8060	-0.8115
100	0.2295	-0.9321	0.2023	-0.8408	0.1815	-0.7750	0.2295	-0.8230	-0.8279
101	0.2468	-0.9479	0.2186	-0.8572	0.1951	-0.7918	0.2468	-0.8382	-0.8424
102	0.2646	-0.9618	0.2354	-0.8733	0.2092	-0.8078	0.2646	-0.8510	-0.8559
103	0.2828	-0.9745	0.2526	-0.8890	0.2235	-0.8227	0.2828	-0.8624	-0.8673
104	0.3013	-0.9868	0.2703	-0.9010	0.2392	-0.8354	0.3013	-0.8727	-0.8769
105	0.3203	-0.9972	0.2884	-0.9128	0.2532	-0.8475	0.3203	-0.8803	-0.8844
106	0.3395	-1.0067	0.3069	-0.9239	0.2684	-0.8587	0.3395	-0.8854	-0.8898
107	0.3591	-1.0160	0.3257	-0.9326	0.2839	-0.8680	0.3591	-0.8910	-0.8920
108	0.3790	-1.0227	0.3449	-0.9401	0.2996	-0.8762	0.3790	-0.8915	-0.8918
109	0.3991	-1.0296	0.3645	-0.9467	0.3155	-0.8827	0.3991	-0.8897	-0.8858
110	0.4194	-1.0349	0.3842	-0.9500	0.3316	-0.8885	0.4194	-0.8829	-0.8746
111	0.4399	-1.0383	0.4043	-0.9523	0.3478	-0.8912	0.4399	-0.8686	-0.8550
112	0.4606	-1.0418	0.4245	-0.9516	0.3641	-0.8924	0.4606	-0.8396	-0.8082
113	0.4814	-1.0417	0.4450	-0.9463	0.3805	-0.8917	0.4814	-0.7298	-0.6388
114	0.5022	-1.0394	0.4656	-0.9371	0.3970	-0.8868	0.5022	-0.4503	-0.3785
115	0.5231	-1.0332	0.4863	-0.9177	0.4136	-0.8792	0.5231	-0.1998	-0.1929
116	0.5441	-1.0200	0.5071	-0.8858	0.4301	-0.8661	0.5441	-0.1311	-0.1426
117	0.5650	-0.9936	0.5279	-0.7801	0.4466	-0.8475	0.5650	-0.1230	-0.1300
118	0.5858	-0.9478	0.5487	-0.4509	0.4631	-0.8012	0.5858	-0.1270	-0.1325
119	0.6065	-0.8572	0.5695	-0.1357	0.4795	-0.6655	0.6065	-0.1288	-0.1356
120	0.6271	-0.4706	0.5903	-0.0554	0.4957	-0.4378	0.6271	-0.1253	-0.1342
121	0.6475	-0.0472	0.6110	-0.0489	0.5224	-0.1946	0.6475	-0.1189	-0.1288
122	0.6677	0.0602	0.6315	-0.0545	0.5498	-0.1289	0.6677	-0.1084	-0.1200
123	0.6876	0.0671	0.6518	-0.0593	0.5768	-0.1308	0.6876	-0.0976	-0.1087
124	0.7073	0.0596	0.6719	-0.0587	0.6034	-0.1359	0.7073	-0.0840	-0.0961
125	0.7266	0.0539	0.6918	-0.0560	0.6296	-0.1340	0.7266	-0.0690	-0.0839
126	0.7455	0.0495	0.7113	-0.0490	0.6552	-0.1265	0.7455	-0.0559	-0.0679
127	0.7640	0.0508	0.7306	-0.0390	0.6803	-0.1133	0.7640	-0.0399	-0.0512
128	0.7821	0.0551	0.7495	-0.0300	0.7048	-0.0977	0.7821	-0.0232	-0.0349
129	0.7997	0.0619	0.7679	-0.0174	0.7286	-0.0827	0.7997	-0.0061	-0.0188
130	0.8168	0.0695	0.7859	-0.0034	0.7517	-0.0625	0.8168	0.0099	-0.0017
131	0.8332	0.0797	0.8034	0.0116	0.7740	-0.0420	0.8332	0.0274	0.0175
132	0.8491	0.0921	0.8204	0.0260	0.7956	-0.0229	0.8491	0.0460	0.0370
133	0.8644	0.1054	0.8368	0.0421	0.8163	-0.0023	0.8644	0.0646	0.0552
134	0.8790	0.1195	0.8527	0.0596	0.8360	0.0207	0.8790	0.0831	0.0742
135	0.8929	0.1342	0.8679	0.0774	0.8548	0.0441	0.8929	0.1016	0.0958
136	0.9060	0.1534	0.8824	0.0955	0.8727	0.0648	0.9060	0.1243	0.1190
137	0.9184	0.1706	0.8962	0.1138	0.8895	0.0898	0.9184	0.1443	0.1391
138	0.9300	0.1887	0.9093	0.1365	0.9052	0.1176	0.9300	0.1650	0.1631
139	0.9408	0.2107	0.9217	0.1570	0.9198	0.1413	0.9408	0.1893	0.1895
140	0.9507	0.2330	0.9333	0.1785	0.9333	0.1700	0.9507	0.2136	0.2172
141	0.9598	0.2563	0.9440	0.2041	0.9456	0.2018	0.9598	0.2306	0.2462
142	0.9680	0.2813	0.9539	0.2306	0.9567	0.2348	0.9680	0.2653	0.2769
143	0.9753	0.3093	0.9630	0.2589	0.9665	0.2701	0.9753	0.2947	0.3098
144	0.9817	0.3398	0.9711	0.2899	0.9751	0.3091	0.9817	0.3266	0.3393
145	0.9871	0.3743	0.9784	0.3243	0.9825	0.3433	0.9871	0.3623	0.3449

Figure A.2- Continued

146	0.9916	0.4036	0.9847	0.3576	0.9886	0.3454	0.9916	0.3927	0.4144
147	0.9951	0.4162	0.9901	0.3694	0.9934	0.4561	0.9951	0.4060	0.4379
148	0.9978	0.4952	0.9946	0.4493	0.9970	0.4186	0.9978	0.4865	0.4186
149	0.9995	0.4947	0.9981	0.4492	0.9993	0.4187	0.9995	0.4865	0.4187

```
***************************
* OUTPUT FOR CASE #2 OF 6 *
***************************

     M2 = 0.8000

     Q2 = 0.1000

     CPCRIT = -0.4346


<<<<< LOCATIONS OF MIN., MAX., AND CRITICAL PTS. >>>>>
  (* DENOTES POINT ON LOWER SURFACE)

     PERTURBATION SOLN:

          MINIMUM AT X = 0.3834     (POINT #112)
          MAXIMUM AT X = 0.0000*    (POINT # 75)
          2 CRITICAL POINT(S):
              1ST AT X = 0.4805*    (AFTER POINT # 41)
              2ND AT X = 0.5229     (AFTER POINT #120)

     COMPARISON SOLN:

          MINIMUM AT X = 0.4015     (POINT #110)
          MAXIMUM AT X = 0.0000     (POINT # 75)
          2 CRITICAL POINT(S):
              1ST AT X = 0.4816*    (AFTER POINT # 36)
              2ND AT X = 0.5258     (AFTER POINT #116)
```

POINT	XBASE	CPBASE	XCALB	CPCALB	XPERT	CPPERT	XCHEK	CPCHEK	CPPERT(INT)
1	0.9995	0.4947	0.9981	0.4492	0.9996	0.4341	0.9989	0.4589	0.4341
2	0.9978	0.4943	0.9946	0.4490	0.9981	0.4341	0.9963	0.4588	0.4557
3	0.9951	0.4144	0.9901	0.3687	0.9958	0.4606	0.9927	0.3781	0.4138
4	0.9916	0.4011	0.9847	0.3567	0.9928	0.4151	0.9882	0.3713	0.3621
5	0.9871	0.3709	0.9784	0.3231	0.9890	0.3632	0.9828	0.3389	0.3483
6	0.9817	0.3354	0.9711	0.2884	0.9843	0.3565	0.9765	0.3055	0.3148
7	0.9753	0.3037	0.9630	0.2570	0.9789	0.3273	0.9692	0.2738	0.2811
8	0.9680	0.2744	0.9539	0.2283	0.9727	0.2957	0.9610	0 2453	0.2503
9	0.9598	0.2478	0.9440	0.2012	0.9657	0.2666	0.9520	0.2188	0.2216
10	0.9507	0.2228	0.9333	0.1750	0.9580	0.2396	0.9420	0.1934	0.1944
11	0.9408	0.1986	0.9217	0.1528	0.9495	0.2142	0.9313	0.1683	0.1692
12	0.9300	0.1743	0.9093	0.1315	0.9403	0.1897	0.9197	0.1470	0.1468
13	0.9 84	0.1535	0.8962	0.1078	0.9304	0.1671	0.9073	0.1264	0.1249
14	0.9060	0.1334	0.8824	0.0884	0.9198	0.1469	0.8942	0.1033	0.1022
15	0.8929	0.1106	0.8679	0.0690	0.9086	0.1271	0.8804	0.0843	0.0828
16	0.8790	0.0918	0.8527	0.0496	0.8967	0.1058	0.8658	0.0653	0.0634
17	0.8644	0.0728	0.8368	0.0302	0.8843	0.0881	0.8506	0.0463	0.0438
18	0.8491	0.0538	0.8204	0.0118	0.8713	0.0705	0.8347	0.0273	0.0246
19	0.8332	0.0346	0.8034	-0.0053	0.8577	0.0528	0.8183	0.0093	0.0071
20	0.8168	0.0163	0.7859	-0.0237	0.8436	0.0350	0.8013	-0.0074	-0.0105
21	0.7997	-0.0009	0.7679	-0.0417	0.8291	0.0180	0.7837	-0.0253	-0.0286
22	0.7821	-0.0193	0.7495	-0.0595	0.8141	0.0029	0.7657	-0.0428	-0.0469
23	0.7640	-0.0377	0.7306	-0.0747	0.7986	-0.0133	0.7472	-0.0598	-0.0623
24	0.7455	-0.0559	0.7113	-0.0926	0.7829	-0.0295	0.7283	-0.0743	-0.0786
25	0.7266	-0.0719	0.6918	-0.1098	0.7667	-0.0461	0.7090	-0.0911	-0.0955

Figure A.2- Continued

26	0.7073	-0.0908	0.6719	-0.1252	0.7502	-0.0598	0.6895	-0.1068	-0.1106
27	0.6876	-0.1094	0.6518	-0.1418	0.7335	-0.0738	0.6696	-0.1204	-0.1257
28	0.6677	-0.1267	0.6315	-0.1564	0.7164	-0.0896	0.6494	-0.1346	-0.1393
29	0.6475	-0.1460	0.6110	-0.1706	0.6992	-0.1034	0.6291	-0.1459	-0.1518
30	0.6271	-0.1643	0.5903	-0.1819	0.6818	-0.1163	0.6085	-0.1556	-0.1598
31	0.6065	-0.1838	0.5695	-0.1905	0.6642	-0.1298	0.5878	-0.1610	-0.1644
32	0.5858	-0.2024	0.5487	-0.1968	0.6466	-0.1412	0.5670	-0.1624	-0.1658
33	0.5650	-0.2213	0.5279	-0.1989	0.6288	-0.1520	0.5462	-0.1625	-0.1641
34	0.5441	-0.2417	0.5071	-0.2033	0.6109	-0.1592	0.5253	-0.1683	-0.1718
35	0.5231	-0.2608	0.4863	-0.2357	0.5931	-0.1637	0.5045	-0.2166	-0.2237
36	0.5022	-0.2786	0.4656	-0.3675	0.5752	-0.1662	0.4836	-0.4092	-0.3977
37	0.4814	-0.2955	0.4450	-0.5861	0.5574	-0.1652	0.4629	-0.6656	-0.6327
38	0.4606	-0.3119	0.4245	-0.7468	0.5397	-0.1634	0.4423	-0.7983	-0.7985
39	0.4399	-0.3264	0.4043	-0.8046	0.5221	-0.1737	0.4218	-0.8384	-0.8483
40	0.4194	-0.3504	0.3842	-0.8226	0.5046	-0.2228	0.4015	-0.8530	-0.8645
41	0.3991	-0.4033	0.3645	-0.8300	0.4872	-0.3556	0.3815	-0.8587	-0.8689
42	0.3790	-0.4976	0.3449	-0.8304	0.4650	-0.6156	0.3616	-0.8609	-0.8689
43	0.3591	-0.6009	0.3257	-0.8280	0.4407	-0.8117	0.3421	-0.8583	-0.8665
44	0.3395	-0.6682	0.3069	-0.8232	0.4166	-0.8583	0.3229	-0.8537	-0.8621
45	0.3203	-0.7003	0.2884	-0.8152	0.3930	-0.8681	0.3040	-0.8472	-0.8546
46	0.3013	-0.7119	0.2703	-0.8057	0.3698	-0.8697	0.2855	-0.8378	-0.8457
47	0.2828	-0.7121	0.2526	-0.7946	0.3470	-0.8675	0.2673	-0.8272	-0.8352
48	0.2646	-0.7073	0.2354	-0.7810	0.3247	-0.8628	0.2496	-0.8151	-0.8225
49	0.2468	-0.6991	0.2186	-0.7654	0.3029	-0.8542	0.2324	-0.8006	-0.8081
50	0.2295	-0.6872	0.2023	-0.7485	0.2817	-0.8439	0.2156	-0.7844	-0.7920
51	0.2127	-0.6726	0.1866	-0.7282	0.2610	-0.8314	0.1993	-0.7669	-0.7735
52	0.1964	-0.6561	0.1714	-0.7074	0.2410	-0.8157	0.1835	-0.7460	-0.7533
53	0.1806	-0.6355	0.1567	-0.6859	0.2216	-0.7987	0.1683	-0.7247	-0.7339
54	0.1653	-0.6140	0.1427	-0.6590	0.2029	-0.7782	0.1536	-0.7027	-0.7095
55	0.1507	-0.5916	0.1292	-0.6298	0.1849	-0.7550	0.1396	-0.6752	-0.6821
56	0.1366	-0.5629	0.1164	-0.5989	0.1676	-0.7330	0.1261	-0.6455	-0.6523
57	0.1231	-0.5308	0.1041	-0.5656	0.1510	-0.7051	0.1132	-0.6140	-0.6204
58	0.1102	-0.4968	0.0926	-0.5280	0.1352	-0.6733	0.1010	-0.5800	-0.5857
59	0.0979	-0.4605	0.0817	-0.4884	0.1202	-0.6386	0.0894	-0.5414	-0.5472
60	0.0863	-0.4184	0.0715	-0.4412	0.1059	-0.6013	0.0785	-0.5003	-0.5047
61	0.0754	-0.3723	0.0619	-0.3945	0.0926	-0.5591	0.0683	-0.4514	-0.4574
62	0.0652	-0.3197	0.0531	-0.3362	0.0800	-0.5117	0.0588	-0.4023	-0.4026
63	0.0557	-0.2688	0.0449	-0.2716	0.0683	-0.4575	0.0500	-0.3402	-0.3403
64	0.0469	-0.2053	0.0375	-0.2113	0.0575	-0.3955	0.0418	-0.2708	-0.2775
65	0.0388	-0.1317	0.0308	-0.1366	0.0476	-0.3229	0.0345	-0.2044	-0.2056
66	0.0314	-0.0567	0.0248	-0.0514	0.0386	-0.2517	0.0278	-0.1217	-0.1194
67	0.0248	0.0373	0.0196	0.0521	0.0305	-0.1608	0.0219	-0.0253	-0.0199
68	0.0190	0.1485	0.0150	0.1636	0.0233	-0.0493	0.0167	0.0935	0.0917
69	0.0139	0.2859	0.0111	0.2966	0.0171	0.0806	0.0122	0.2265	0.2268
70	0.0096	0.4435	0.0078	0.4490	0.0118	0.2404	0.0085	0.3875	0.3973
71	0.0061	0.6300	0.0051	0.6219	0.0075	0.4458	0.0054	0.5727	0.5947
72	0.0034	0.8314	0.0030	0.8106	0.0041	0.6902	0.0031	0.7749	0.7999
73	0.0015	1.0193	0.0014	0.9949	0.0018	0.9348	0.0014	0.9718	0.9890
74	0.0004	1.1454	0.0004	1.1296	0.0004	1.1133	0.0004	1.1201	1.1243
75	0.0000	1.1617	0.0000	1.1692	0.0000	1.1716	0.0000	1.1701	1.1715
76	0.0004	1.0535	0.0004	1.0967	0.0003	1.1228	0.0004	1.1034	1.1138
77	0.0015	0.8493	0.0014	0.9332	0.0012	0.9825	0.0014	0.9406	0.9603
78	0.0034	0.6062	0.0030	0.7278	0.0028	0.7753	0.0031	0.7335	0.7474
79	0.0061	0.3725	0.0051	0.5262	0.0051	0.5599	0.0054	0.5249	0.5349
80	0.0096	0.1683	0.0078	0.3464	0.0080	0.3680	0.0085	0.3363	0.3447
81	0.0139	0.0054	0.0111	0.1907	0.0116	0.2026	0.0122	0.1738	0.1820
82	0.0190	-0.1336	0.0150	0.0568	0.0158	0.0677	0.0167	0.0407	0.0469
83	0.0248	-0.2430	0.0196	-0.0540	0.0207	-0.0466	0.0219	-0.0700	-0.0689
84	0.0314	-0.3358	0.0248	-0.1570	0.0262	-0.1470	0.0278	-0.1738	-0.1694
85	0.0388	-0.4104	0.0308	-0.2415	0.0323	-0.2308	0.0344	-0.2562	-0.2526

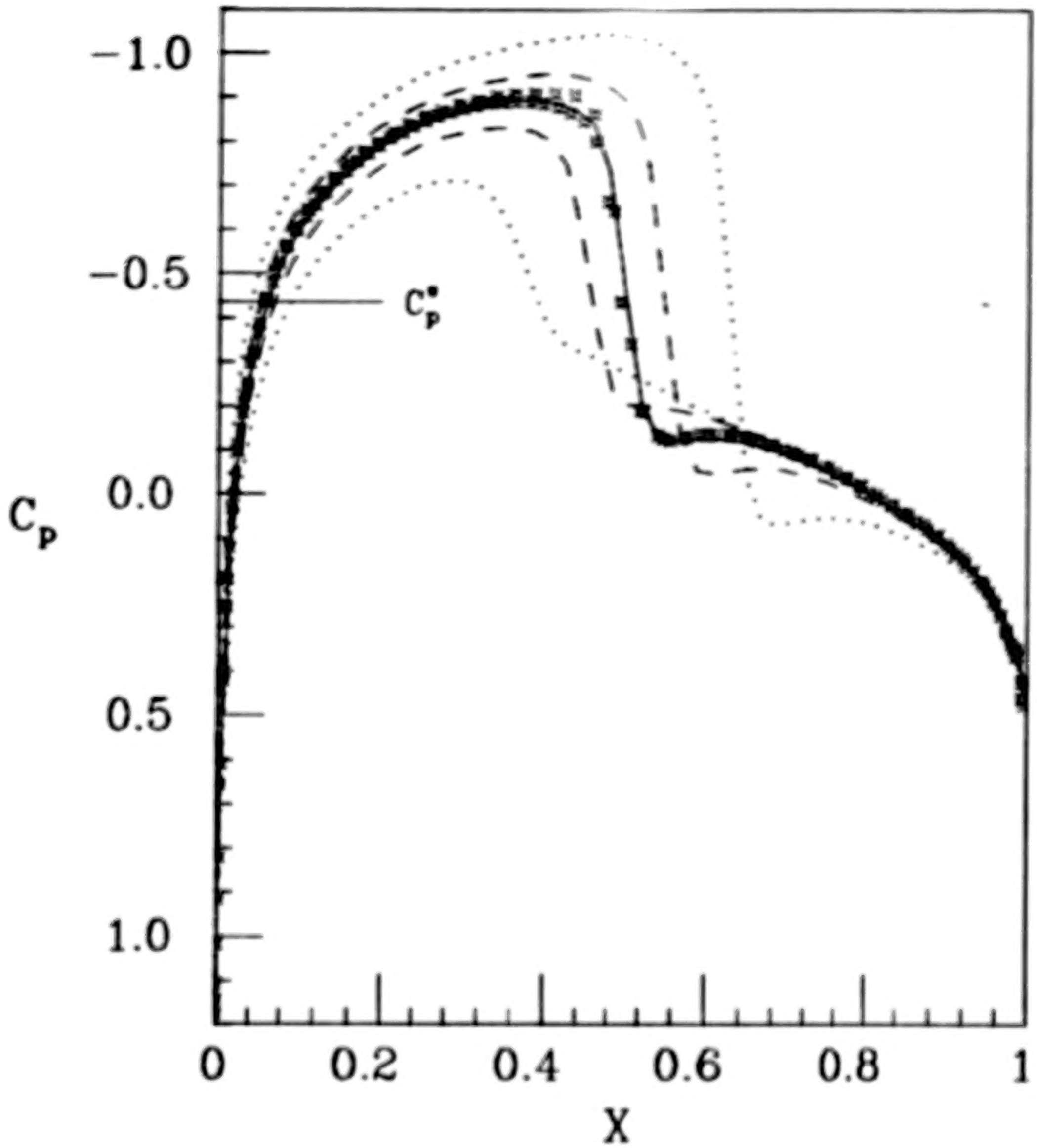
Figure A.2- Continued

86	0.0469	-0.4834	0.0375	-0.3160	0.0390	-0.2986	0.0418	-0.3229	-0.3231
87	0.0557	-0.5434	0.0449	-0.3774	0.0463	-0.3627	0.0500	-0.3936	-0.3928
88	0.0652	-0.5903	0.0531	-0.4440	0.0543	-0.4285	0.0588	-0.4554	-0.4545
89	0.0754	-0.6388	0.0619	-0.5002	0.0628	-0.4779	0.0683	-0.5019	-0.5040
90	0.0863	-0.6794	0.0715	-0.5425	0.0719	-0.5208	0.0785	-0.5492	-0.5505
91	0.0979	-0.7167	0.0817	-0.5880	0.0815	-0.5638	0.0894	-0.5899	-0.5930
92	0.1102	-0.7492	0.0926	-0.6268	0.0917	-0.6015	0.1010	-0.6277	-0.6309
93	0.1231	-0.7793	0.1041	-0.6626	0.1024	-0.6355	0.1132	-0.6607	-0.6642
94	0.1366	-0.8075	0.1164	-0.6940	0.1137	-0.6654	0.1261	-0.6913	-0.6949
95	0.1507	-0.8341	0.1292	-0.7233	0.1254	-0.6933	0.1396	-0.7202	-0.7250
96	0.1653	-0.8557	0.1427	-0.7512	0.1376	-0.7210	0.1536	-0.7473	-0.7519
97	0.1806	-0.8763	0.1567	-0.7775	0.1503	-0.7469	0.1683	-0.7693	-0.7740
98	0.1[illegible]64	-0.8968	0.1714	-0.7989	0.1635	-0.7669	0.1835	-0.7906	-0.7959
99	0.2127	-0.9149	0.1866	-0.8197	0.1771	-0.7868	0.1993	-0.8114	-0.8159
100	0.2295	-0.9321	0.2023	-0.8400	0.1911	-0.8064	0.2156	-0.8291	-0.8337
101	0.2468	-0.9479	0.2186	-0.8572	0.2055	-0.8230	0.2324	-0.8457	-0.8502
102	0.2646	-0.9618	0.2354	-0.8733	0.2202	-0.8386	0.2496	-0.8607	-0.8647
103	0.2828	-0.9745	0.2526	-0.8880	0.2354	-0.8530	0.2673	-0.8734	-0.8779
104	0.3013	-0.9868	0.2703	-0.9010	0.2508	-0.8657	0.2855	-0.8849	-0.8899
105	0.3203	-0.9972	0.2884	-0.9128	0.2666	-0.8774	0.3040	-0.8958	-0.9000
106	0.3395	-1.0067	0.3069	-0.9239	0.2826	-0.8883	0.3229	-0.9043	-0.9084
107	0.3591	-1.0160	0.3257	-0.9326	0.2989	-0.8976	0.3421	-0.9114	-0.9154
108	0.3790	-1.0227	0.3449	-0.9401	0.3155	-0.9055	0.3616	-0.9173	-0.9198
109	0.3991	-1.0296	0.3645	-0.9467	0.3322	-0.9121	0.3815	-0.9194	-0.9221
110	0.4194	-1.0349	0.3842	-0.9500	0.3491	-0.9178	0.4015	-0.9200	-0.9215
111	0.4399	-1.0383	0.4043	-0.9523	0.3662	-0.9206	0.4218	-0.9166	-0.9157
112	0.4606	-1.0418	0.4245	-0.9516	0.3834	-0.9223	0.4423	-0.9075	-0.9049
113	0.4814	-1.0417	0.4450	-0.9463	0.4007	-0.9217	0.4629	-0.8922	-0.8853
114	0.5022	-1.0394	0.4656	-0.9371	0.4181	-0.9173	0.4836	-0.8608	-0.8411
115	0.5231	-1.0332	0.4863	-0.9177	0.4355	-0.9100	0.5045	-0.7489	-0.7069
116	0.5441	-1.0200	0.5071	-0.8858	0.4529	-0.8969	0.5253	-0.4408	-0.4081
117	0.5650	-0.9936	0.5279	-0.7801	0.4703	-0.8767	0.5462	-0.1637	-0.1787
118	0.5858	-0.9478	0.5487	-0.4509	0.4876	-0.8305	0.5670	-0.0936	-0.1091
119	0.6065	-0.8572	0.5695	-0.1357	0.5049	-0.7038	0.5878	-0.0883	-0.0912
120	0.6271	-0.4706	0.5903	-0.0554	0.5220	-0.4444	0.6085	-0.0938	-0.0933
121	0.6475	-0.0472	0.6110	-0.0489	0.5474	-0.1651	0.6291	-0.0962	-0.0967
122	0.6677	0.0602	0.6315	-0.0545	0.5733	-0.0911	0.6494	-0.0951	-0.0963
123	0.6876	0.0671	0.6518	-0.0593	0.5990	-0.0912	0.6696	-0.0890	-0.0921
124	0.7073	0.0596	0.6719	-0.0587	0.6242	-0.0968	0.6895	-0.0816	-0.0840
125	0.7266	0.0539	0.6918	-0.0560	0.6490	-0.0964	0.7090	-0.0707	-0.0736
126	0.7455	0.0495	0.7113	-0.0490	0.6[illegible]33	-0.0913	0.7283	-0.0577	-0.0624
127	0.7640	0.0508	0.7306	-0.0390	0.6970	-0.0805	0.7472	-0.0463	-0.0502
128	0.7821	0.0551	0.7495	-0.0300	0.7202	-0.0672	0.7657	-0.0316	-0.0353
129	0.7997	0.0619	0.7679	-0.0174	0.7428	-0.0537	0.7837	-0.0160	-0.0195
130	0.8168	0.0695	0.7859	-0.0034	0.7647	-0.0361	0.8013	0.0003	-0.0043
131	0.8332	0.0797	0.8034	0.0116	0.7859	-0.0176	0.8183	0.0157	0.0118
132	0.8491	0.0921	0.8204	0.0260	0.8063	0.0001	0.8347	0.0327	0.0293
133	0.8644	0.1054	0.8368	0.0421	0.8259	0.0193	0.8506	0.0508	0.0477
134	0.8790	0.1195	0.8527	0.0596	0.8446	0.0405	0.8658	0.0691	0.0662
135	0.8929	0.1342	0.8679	0.0774	0.8624	0.0621	0.8804	0.0875	0.0841
136	0.9060	0.1534	0.8824	0.0955	0.8793	0.0825	0.8942	0.1060	0.1045
137	0.9184	0.1706	0.8962	0.1138	0.8953	0.1060	0.9073	0.1286	0.1270
138	0.9300	0.1887	0.9093	0.1365	0.9101	0.1318	0.9197	0.1489	0.1480
139	0.9408	0.2107	0.9217	0.1570	0.9240	0.1552	0.9313	0.1699	0.1709
140	0.9507	0.2330	0.9333	0.1785	0.9368	0.1826	0.9420	0.1947	0.1963
141	0.9598	0.2563	0.9440	0.2041	0.9484	0.2127	0.9520	0.2199	0.2233
142	0.9680	0.2813	0.9539	0.2306	0.9589	0.2441	0.9610	0.2462	0.2517
143	0.9753	0.3093	0.9630	0.2589	0.9683	0.2780	0.9692	0.2746	0.2822
144	0.9817	0.3398	0.9711	0.2899	0.9764	0.3153	0.9765	0.3061	0.3154
145	0.9871	0.3743	0.9784	0.3243	0.9834	0.3495	0.9828	0.3394	0.3465
146	0.9916	0.4036	0.9847	0.3576	0.9892	0.3571	0.9882	0.3717	0.3558
147	0.9951	0.4162	0.9901	0.3694	0.9937	0.4481	0.9927	0.3784	0.4278
148	0.9978	0.4952	0.9946	0.4493	0.9971	0.4339	0.9963	0.4590	0.4375
149	0.9995	0.4947	0.9981	0.4492	0.9994	0.4339	0.9989	0.4589	0.4339

Figure A.2- Concluded

Plot of C_p

Full ϕ_{Holst} Nonl. α Pert. $\alpha_b = 0.500$ $\alpha_c = 0.200$



$M_\infty = 0.800$ $\alpha = 0.000$

Figure A.3- Plot output for sample case

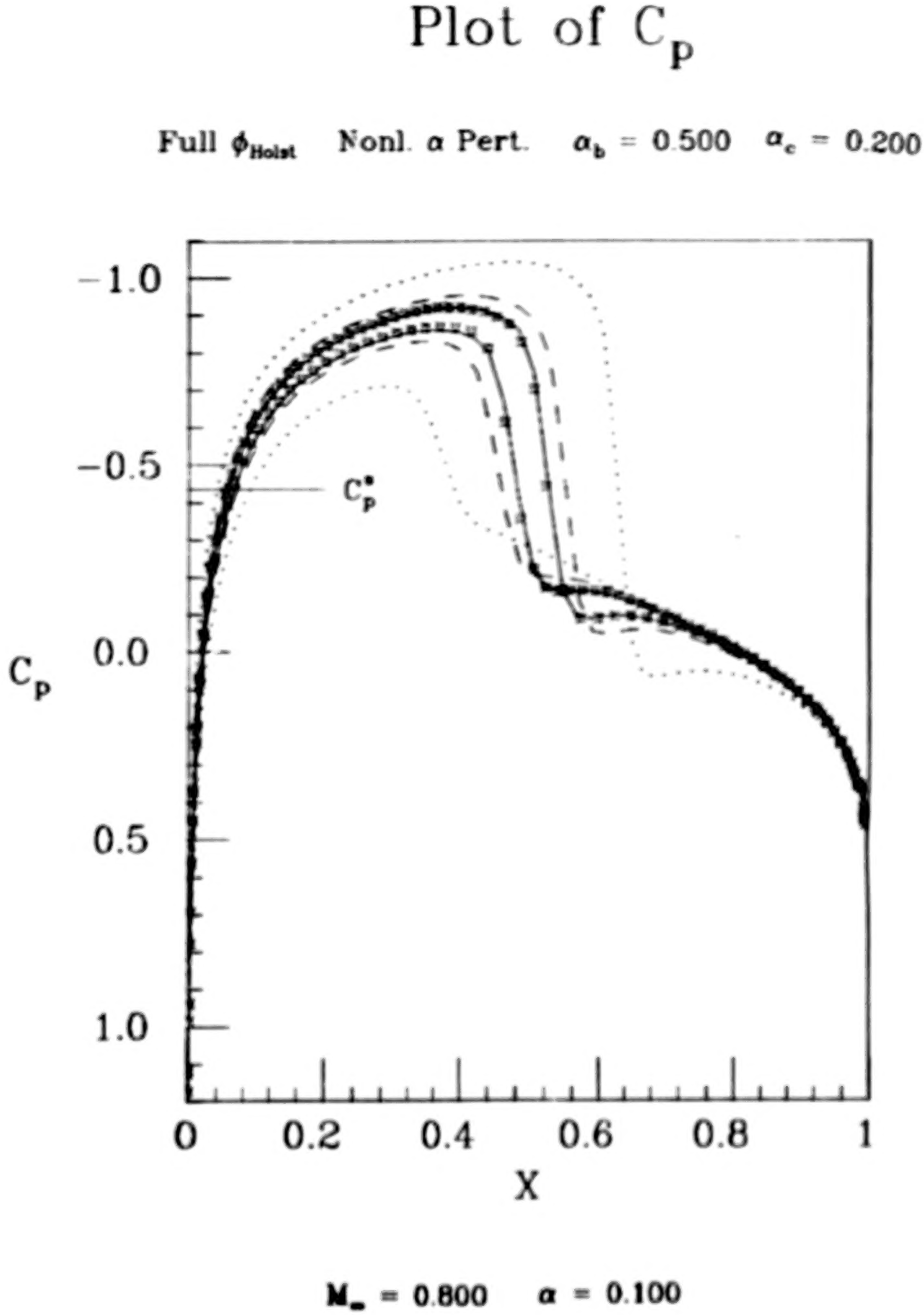
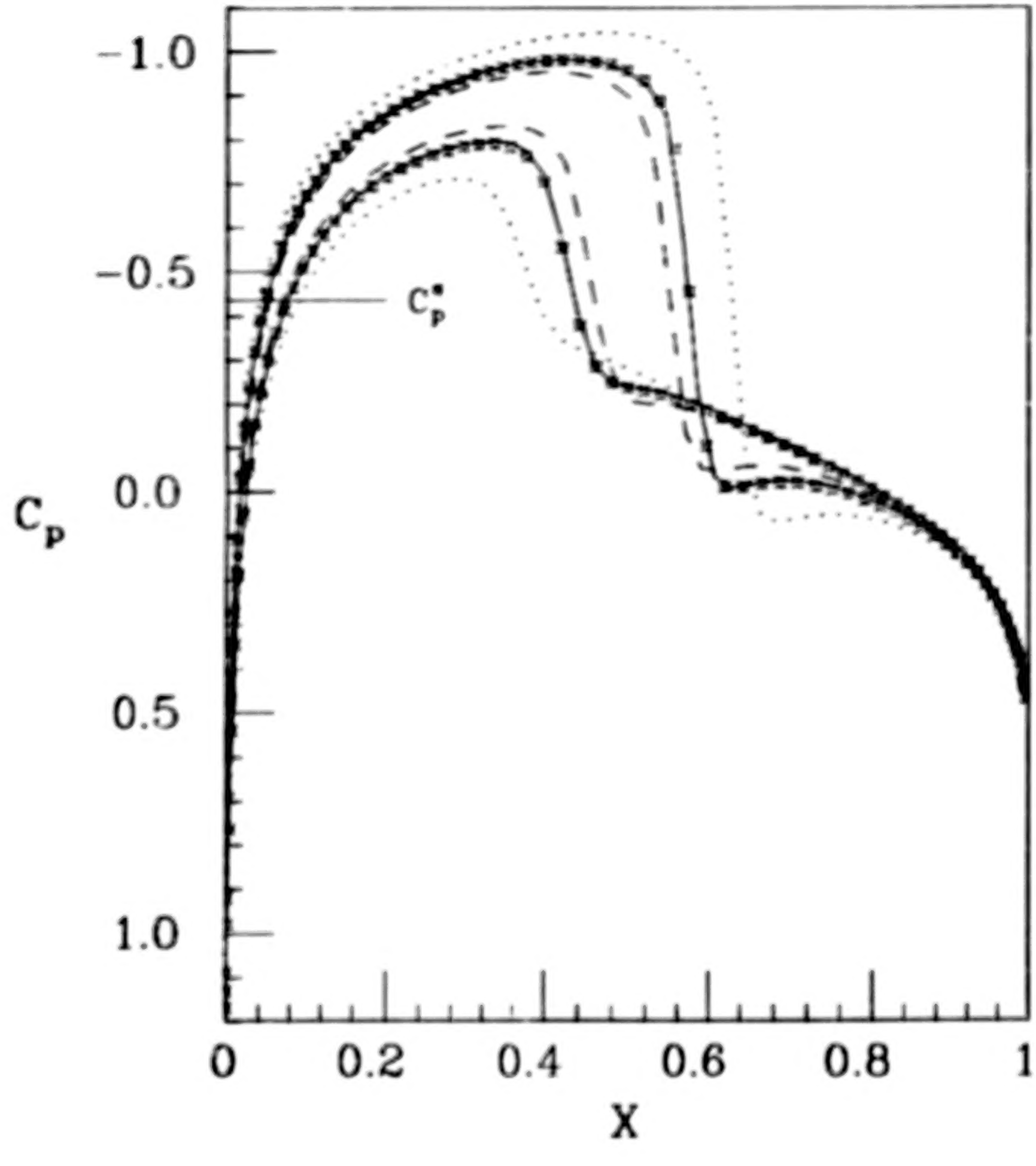


Figure A.3- Continued

Plot of C_p

Full ϕ_{Holst} Nonl. α Pert. $\alpha_b = 0.500$ $\alpha_c = 0.200$



$M_\infty = 0.800$ $\alpha = 0.300$

Figure A.3- Continued

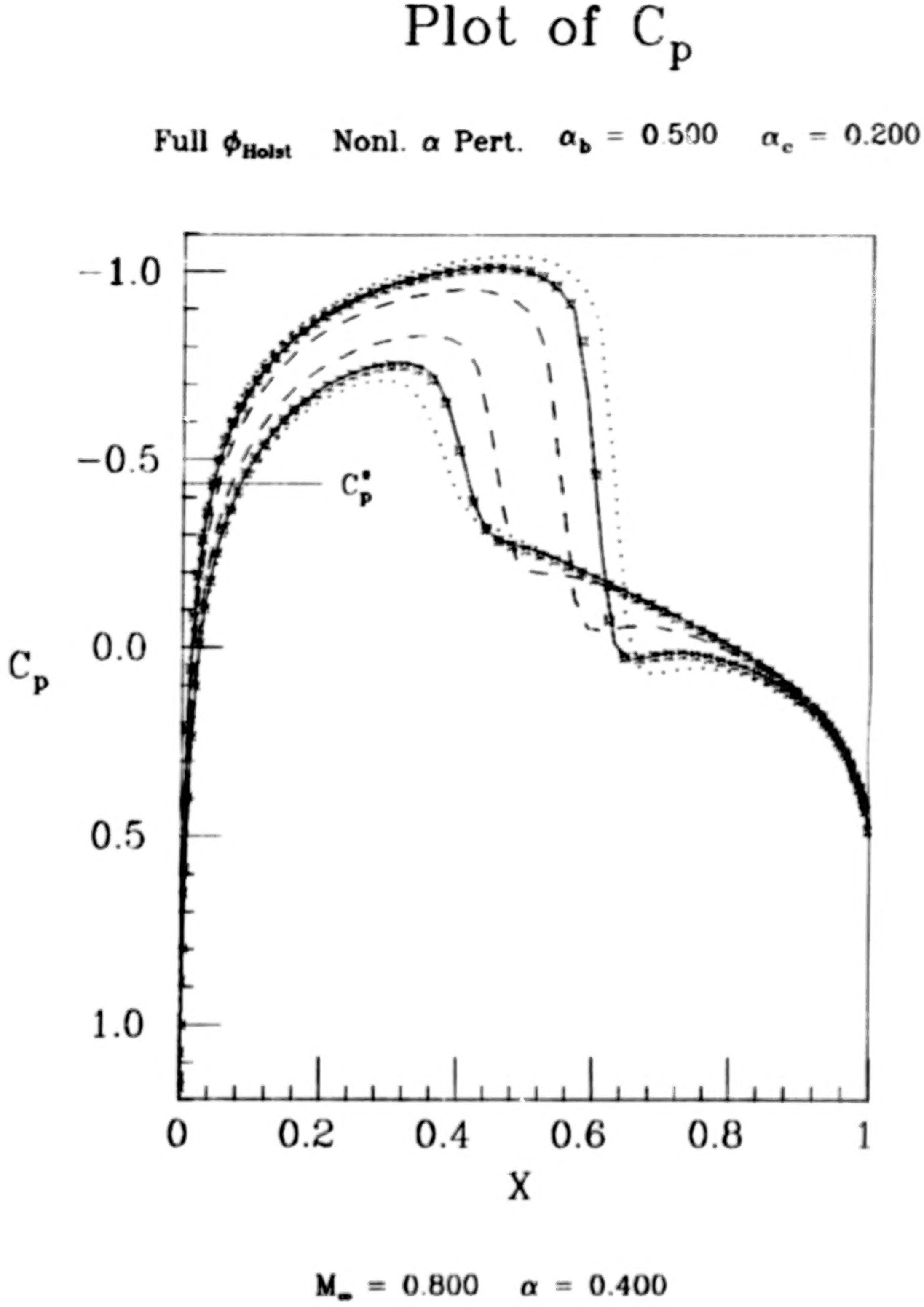


Figure A.3- Continued

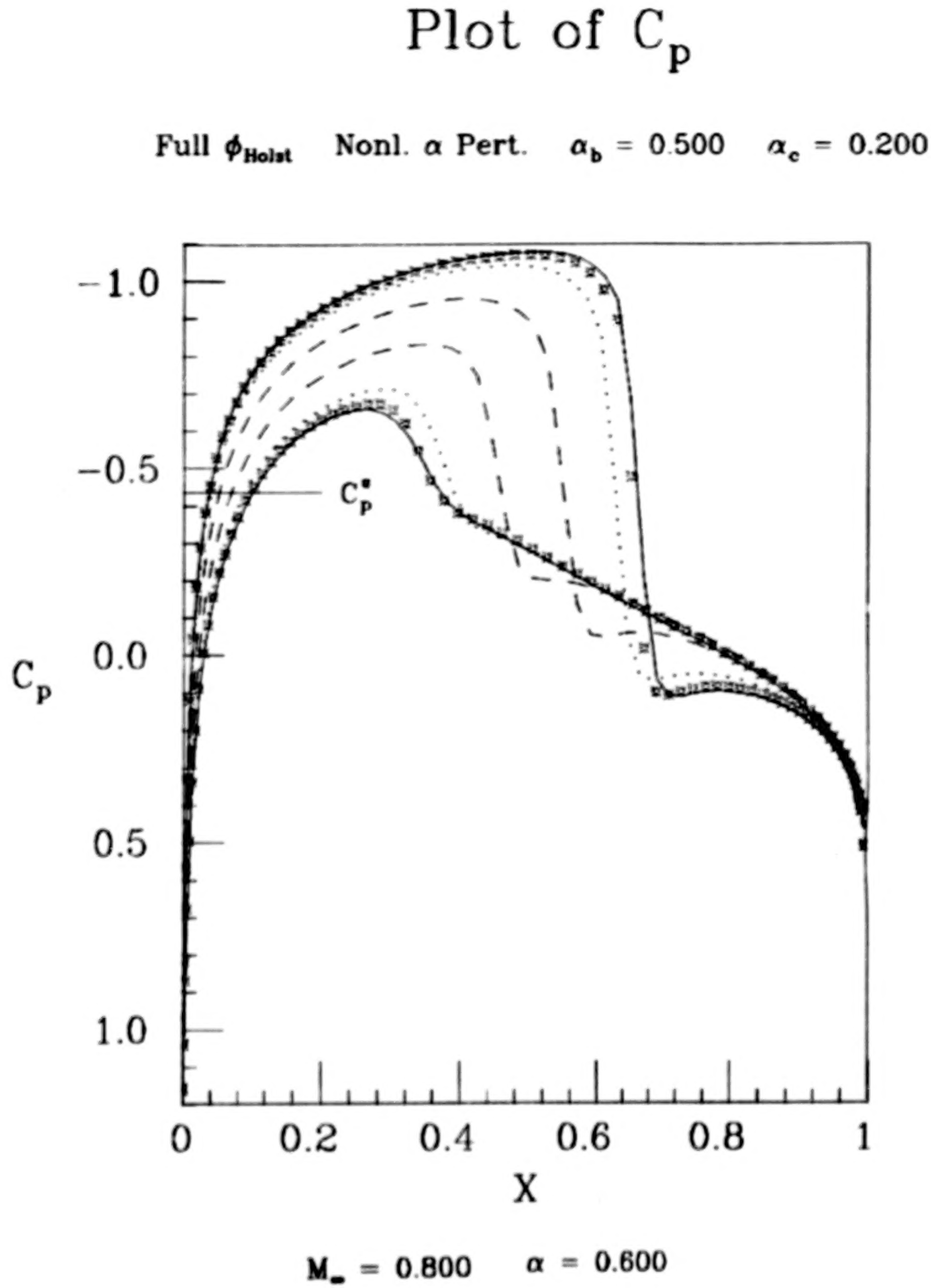
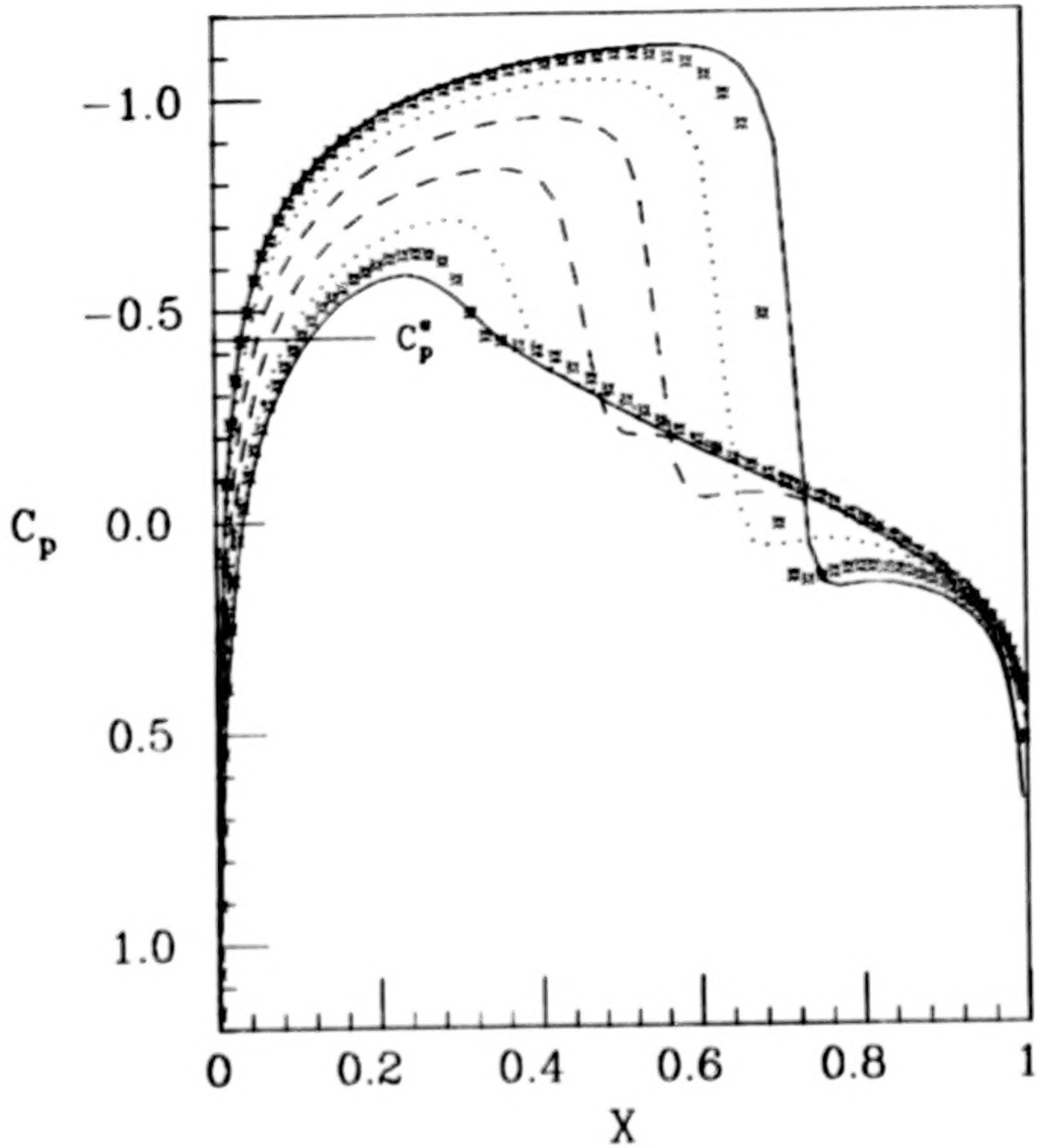


Figure A.3- Continued

Plot of C_p

Full ϕ_{Holst} Nonl. α Pert. $\alpha_b = 0.500$ $\alpha_c = 0.200$



$M_\infty = 0.800$ $\alpha = 0.700$

Figure A.3- Concluded

```
C<<<<<<<<<<<<<<<<<<<<<<<<< MAIN PROGRAM PERTURB >>>>>>>>>>>>>>>>>>>>>>>>>>
C
C     CALCULATES CONTINUOUS OR DISCONTINUOUS NONLINEAR PERTURBATION
C     SOLUTIONS WHICH REPRESENT A SINGLE-PARAMETER CHANGE IN EITHER
C     GEOMETRY OR FLOW CONDITIONS BY EMPLOYING A STRAINED-COORDINATE
C     PROCEDURE. THE METHOD UTILIZES A UNIT PERTURBATION, DETERMINED
C     FROM TWO PREVIOUSLY CALCULATED SOLUTIONS ('BASE' AND 'CALIBRATION'
C     SOLUTIONS) OBTAINED FROM AN 'EXPENSIVE' COMPUTATIONAL PROCEDURE
C     AND DISPLACED FROM ONE ANOTHER BY SOME REASONABLE CHANGE IN
C     GEOMETRY OR FLOW VARIABLE, TO PREDICT NEW NONLINEAR SOLUTIONS OVER
C     A RANGE OF PARAMETER VARIATION.
C
C     THIS CURRENT VERSION OF THE PROCEDURE IS CONFIGURED TO PREDICT AND
C     PLOT PRESSURE COEFFICIENTS ON A BLADE OR AIRFOIL SURFACE, AND CAN
C     ACCOUNT FOR THE MOTION OF:
C                 (1) ONE OR MORE CRITICAL POINTS (SHOCK POINTS),
C                 (2) A STAGNATION POINT,
C                 (3) A MAXIMUM-SUCTION-PRESSURE POINT,
C     OR SIMULTANEOUSLY FOR ANY COMBINATION OF THESE.
C
C     THE PROGRAM IS ALSO CONFIGURED TO COMPARE THE PERTURBATION-
C     PREDICTED SOLUTIONS WITH THE CORRESPONDING 'EXACT' SOLUTIONS
C     OBTAINED BY EMPLOYING THE SAME 'EXPENSIVE' COMPUTATIONAL
C     PROCEDURE USED TO DETERMINE THE BASE AND CALIBRATION SOLUTIONS.
C     SEE THE SUBROUTINE INPUT FOR DETAILS.
C
C     N = NO. OF POINTS IN SURFACE PRESSURE DISTRIBUTION - ASSUMED EQUAL
C     FOR BASE, CALIBRATION, AND PREDICTED DISTRIBUTIONS.
C                      NOTE: N <= 200.
C
C                               BASE              CALIBRATION
C     ONCOMING MACH NO.          M0                   M1
C     PARAMETER PERTURBED        Q0                   Q1
C
C     M2 = ONCOMING MACH NO. OF PREDICTED FLOW
C     Q2 = VALUE OF PERTURBED PARAMETER IN PREDICTED FLOW
C
C     COORDINATE STRAINING IS PIECEWISE LINEAR WITH END POINTS AND ONE
C     OR MORE USER-SELECTED INTERIOR POINTS HELD INVARIANT.
C
C     THE PROGRAM LOCATES MINIMUM, MAXIMUM, AND ALL CRITICAL POINTS
C     (SHOCK POINTS) IN THE BASE AND CALIBRATION SOLUTIONS, AND STORES
C     THESE IN THE ARRAYS XLOC0 AND XLOC1 (IT IS ASSUMED THAT THE NUMBER
C     OF CRITICAL POINTS DOES NOT EXCEED FOUR) AS FOLLOWS:
C
C             BASE                               CALIBRATION
C
C        XLOC0(1) = MINIMUM PT.              XLOC1(1) = MINIMUM PT.
C        XLOC0(2) = MAXIMUM PT.              XLOC1(2) = MAXIMUM PT.
C        XLOC0(3) = CRITICAL PT. #1          XLOC1(3) = CRITICAL PT. #1
C        ...      = ...                      ...      = ...
C        XLOC0(6) = CRITICAL PT. #4          XLOC1(6) = CRITICAL PT. #4
C
C     THE NUMBER OF POINTS SELECTED FROM THESE IS SPECIFIED BY NSELCT.
C     THE CORRESPONDING SUBSCRIPTS OF XLOC0 AND XLOC1 ARE SPECIFIED IN
C     THE FIRST NSELCT ELEMENTS OF THE ARRAY LSECT, E.G. TO SELECT THE
C     MAXIMUM POINT AND THE FIRST AND THIRD CRITICAL POINTS, ONE
C     SPECIFIES:
C
C                         NSELCT = 3
C                         LSELCT(1) = 2
C                         LSELCT(2) = 3
C                         LSELCT(3) = 5
C
C     PROVISION IS ALSO MADE TO ALLOW THE USER TO SPECIFY THE POINTS IN
C     THE BASE AND CALIBRATION SOLUTIONS THAT WILL BE HELD INVARIANT, TO
C     A MAXIMUM OF SIX. SEE THE SUBROUTINE INPUT FOR DETAILS.
C
      DIMENSION XBASE(200),XCALB(200),XCHEK(200),XPERT(200),XUNIT(200)
      DIMENSION YBASE(200),YCALB(200),YPERT(200),YCHEK(200),YINTP(200),
     X          YPRTI(200),YUNIT(200)
      DIMENSION XLOC0(6),XLOC1(6),XLOC2(6),XLOC3(6),XFIX0(8),XFIX1(8)
      DIMENSION LCR0(4),LCR1(4),LCR2(4),LCR3(4),LOC0(6),LOC1(6),
     X          ISEQ0(8),ISEQ1(8),LSELCT(6)
      DIMENSION XOUT(8)
      DIMENSION TITLE(20)
      INTEGER*2 NAME
      INTEGER HEAD0(5) /4HBASE,4H SOL,4HUTIO,4HN:  ,4H    /,
     X        HEAD1(5) /4HCALI,4HBRAT,4HION ,4HSOLN,4H:   /,
     X        HEAD2(5) /4HPERT,4HURBA,4HTION,4H SOL,4HN:  /,
     X        HEAD3(5) /4HCOMP,4HARIS,4HON S,4HOLN:,4H    /
      INTEGER ORD(4) /4H1ST ,4H2ND ,4H3RD ,4H4TH /
      LOGICAL*1 FLAG(8)
      REAL M0,M1,M2
      COMMON /COEFF/ C(7),D(7)
      COMMON /PARAM/ TITLE,LOC0,LOC1,LSELCT,N,NCASE,LSPEC,LECHO,LUNIT,
     X               LCHEK,LPERT,NSELCT,A,B,NAME
      COMMON /PERT/ M0,M1,M2,Q0,Q1,Q2,YCR0,YCR1,YCR2
      COMMON /XY/ XBASE,XCALB,XPERT,XCHEK,YBASE,YCALB,YPERT,YCHEK
      DATA LTERM /0/, LCORR /0/
C
C*************************************************************************
C
C     USER-SUPPLIED STATEMENT FUNCTION YCRIT(Z) DETERMINES CRITICAL
C     VALUES OF FLOW VARIABLE YCRIT AS FUNCTION OF FLOW PARAMETER Z.
C     IGRAD (+1 OR -1) IS THE USER-SUPPLIED ALGEBRAIC SIGN OF DYCRIT/DX
C     USED IN LOCATING THE CRITICAL POINT.
C
C     IN THE PRESENT VERSION OF THE CODE, YCRIT REPRESENTS THE FULL-
C     POTENTIAL CRITICAL PRESSURE COEFFICIENT FOR AIR (GAMMA = 1.4), Z
C     IS THE FREE STREAM MACH NUMBER, AND IGRAD CORRESPONDS TO POSITIVE
C     PRESSURE GRADIENT (+1).
C
      YCRIT(Z)=2.0*(((2.0+0.4*Z**2)/2.4)**(1.4/0.4)-1.0)/(1.4*Z**2)
      IGRAD=1
C
C*************************************************************************
C
C.....INPUT CONTROL, GEOMETRY, AND STRAINING PARAMETERS, AND DATA FOR
C     BASE AND CALIBRATION SOLUTIONS.
C
      CALL INPUT(1)
C
C.....PRINT BANNER PAGE
C
      CALL BANNER
C
C.....ECHO INPUT DECK IF LECHO .NE. 0.
C
      IF (LECHO .NE. 0) CALL ECHINP
```

```
C
      DELT=Q1-Q0
      YCR0=YCRIT(M0)
      YCR1=YCRIT(M1)
      WRITE (6,2000) TITLE
      WRITE (6,2010) N,A,B,HEAD0,M0,Q0,HEAD1,M1,Q1
      WRITE (6,2020) NAME,HEAD0,NAME,YCR0,HEAD1,NAME,YCR1
C
C.....NORMALIZE X COORDINATES AND LOCATE MINIMUM, MAXIMUM, AND CRITICAL
C     POINTS FOR BASE AND CALIBRATION SOLUTIONS.
C
      CALL SCALE (N,XBASE,1,A,B)
      CALL LOCATE (N,XBASE,YBASE,YCR0,IGRAD,LMN0,LMX0,NCR0,LCR0,XLOC0)
      CALL SCALE (N,XCALB,1,A,B)
      CALL LOCATE (N,XCALB,YCALB,YCR1,IGRAD,LMN1,LMX1,NCR1,LCR1,XLOC1)
      WRITE (6,2030)
      WRITE (6,2035)
      CALL UPLOW (A,B,XLOC0,6,NCR0+2,XOUT,FLAG)
      WRITE (6,2040) HEAD0,XOUT(1),FLAG(1),LMN0,XOUT(2),FLAG(2),LMX0
      IF (NCR0 .GT. 0) WRITE (6,2045) NCR0,
     Z     (ORD(I),XOUT(I+2),FLAG(I+2),LCR0(I),I=1,NCR0)
      CALL UPLOW (A,B,XLOC1,6,NCR1+2,XOUT,FLAG)
      WRITE (6,2040) HEAD1,XOUT(1),FLAG(1),LMN1,XOUT(2),FLAG(2),LMX1
      IF (NCR1 .GT. 0) WRITE (6,2045) NCR1,
     Z     (ORD(I),XOUT(I+2),FLAG(I+2),LCR1(I),I=1,NCR1)
C
C.....CHECK FOR INVALID STRAINING SPECIFICATION IF LSPEC = 0.
C
      IF (LSPEC .EQ. 1) GO TO 4
      ICOUNT=0
      DO 2 I=1,NSELCT
      IF (LSELCT(I) .LE. 2) GO TO 2
      ICOUNT=ICOUNT+1
      IF (NCR0 .NE. NCR1) LTERM=1
    2 CONTINUE
C
C.....STOP EXECUTION IF CRITICAL POINTS ARE TO BE USED IN STRAINING AND
C     NUMBER OF CRITICAL POINTS IN BASE AND CALIBRATION SOLUTIONS ARE
C     UNEQUAL.
C
      IF (LTERM .EQ. 1) GO TO 900
C
C.....STOP EXECUTION IF NUMBER OF CRITICAL POINTS SELECTED EXCEEDS
C     NUMBER ACTUALLY LOCATED.
C
      IF (ICOUNT .GT. NCR0) GO TO 905
C
    4 CONTINUE
C
C.....LOAD SELECTED STRAINING POINTS INTO FIXED-POINT ARRAYS FOR BASE
C     AND CALIBRATION SOLUTIONS.
C
      NFIX=NSELCT+2
      XFIX0(1)=0.0
      XFIX1(1)=0.0
      XFIX0(NFIX)=1.0
      XFIX1(NFIX)=1.0
      DO 10 I=1,NSELCT
      IF (LSPEC .EQ. 0) GO TO 5
      XFIX0(I+1)=XBASE(LOC0(I))
      XFIX1(I+1)=XCALB(LOC1(I))
      GO TO 10
    5 CONTINUE
      XFIX0(I+1)=XLOC0(LSELCT(I))
      XFIX1(I+1)=XLOC1(LSELCT(I))
   10 CONTINUE
      WRITE (6,2050) NFIX
      IF (LSPEC .EQ. 0) GO TO 14
      WRITE (6,2046) HEAD0,HEAD1
      DO 13 I=1,NSELCT
      WRITE (6,2047) LOC0(I),LOC1(I)
   13 CONTINUE
      GO TO 16
   14 CONTINUE
      DO 15 I=1,NSELCT
      IF (LSELCT(I) .EQ. 1) WRITE (6,2060) NAME
      IF (LSELCT(I) .EQ. 2) WRITE (6,2070) NAME
      IF (LSELCT(I) .LE. 2) GO TO 15
      LCORR=1
      LPR=LSELCT(I)-2
      WRITE (6,2080) NAME,ORD(LPR),NCR0
   15 CONTINUE
   16 CONTINUE
C
C.....ARRANGE SELECTED FIXED POINTS IN A MONOTONE SEQUENCE.
C
      CALL SORT (NFIX,XFIX0,ISEQ0)
      CALL SORT (NFIX,XFIX1,ISEQ1)
      WRITE (6,2090)
      WRITE (6,2035)
      CALL UPLOW (A,B,XFIX0,6,NFIX,XOUT,FLAG)
      WRITE (6,2100) HEAD0,(I,XOUT(I),FLAG(I),I=1,NFIX)
      CALL UPLOW (A,B,XFIX1,6,NFIX,XOUT,FLAG)
      WRITE (6,2100) HEAD1,(I,XOUT(I),FLAG(I),I=1,NFIX)
C
C.....STOP EXECUTION IF ORDER OF OCCURRENCE OF CRITICAL POINTS IN BASE
C     AND CALIBRATION SOLUTIONS DOES NOT CORRESPOND.
C
      IF (LSPEC .EQ. 1) GO TO 25
      DO 20 I=1,NFIX
      IF (ISEQ0(I) .NE. ISEQ1(I)) GO TO 910
   20 CONTINUE
   25 CONTINUE
C
C.....COMPUTE COEFFICIENTS IN UNIT STRAINING OF XBASE
C
C           XSTR = C(I) + D(I)*XBASE,  I=1,2, ... ,NSEG,
C
C     WHERE NSEG IS THE NUMBER OF LINEAR SEGMENTS.
C
      NSEG=NFIX-1
      DO 30 I=1,NSEG
      CNUM=XFIX1(I)*XFIX0(I+1)-XFIX1(I+1)*XFIX0(I)
      DNUM=XFIX1(I+1)-XFIX1(I)
      DENOM=XFIX0(I+1)-XFIX0(I)
      C(I)=CNUM/DENOM
      D(I)=DNUM/DENOM
   30 CONTINUE
C
C.....DETERMINE UNIT STRAINING OF XBASE
```

8

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C
      CALL STRAIN (N,NSEG,XFIXD,XBASE,1.0,XUNIT)
C
C.....INTERPOLATE CALIBRATION SOLUTION TO BASE FLOW POINTS CORRESPONDING
C     TO UNIT STRAINING.
C
      CALL INTERP (N,XCALB,YCALB,XUNIT,YINTP)
C
C.....CORRECT VALUES ON EITHER SIDE OF CRITICAL POINTS, IF THESE ARE
C     USED IN STRAINING.
C
      IF (LCORR .EQ. 0) GO TO 36
      DO 35 I=1,NCR1
      YINTP(LCRD(I))=YCALB(LCR1(I))
      YINTP(LCRD(I)+1)=YCALB(LCR1(I)+1)
   35 CONTINUE
   36 CONTINUE
C
C.....RESTORE PHYSICAL X IN CALIBRATION SOLUTION SINCE IT WILL NOT BE
C     FURTHER USED.
C
      CALL SCALE (N,XCALB,2,A,B)
C
C.....DETERMINE THE UNIT PERTURBATION.
C
      DO 40 I=1,N
   40 YUNIT(I)=(YINTP(I)-YBASE(I))/DEL1
C
C.....PRINT UNIT PERTURBATION AND UNIT STRAINING IF LUNIT .NE. 0.
C
      IF (LUNIT .EQ. 0) GO TO 50
      CALL SCALE (N,XBASE,2,A,B)
      CALL SCALE (N,XUNIT,2,A,B)
      WRITE (6,2110) NAME,NAME
      WRITE (6,2120) (I,XBASE(I),XUNIT(I),YUNIT(I),I=1,N)
      CALL SCALE (N,XBASE,1,A,B)
   50 CONTINUE
C
C.....CONSTRUCT PERTURBATION SOLUTIONS FOR TEST CASES (AND COMPARE WITH
C     EXACT SOLUTION, IF AVAILABLE).
C
      DO 200 ICASE=1,NCASE
      CALL INPUT(2)
      DEL2=Q2-Q0
      DEL21=DEL2/DEL1
      YCR2=YCRIT(M2)
      YCR3=YCR2
C
C.....DETERMINE STRAINED COORDINATE FOR GIVEN PERTURBATION.
C
      CALL STRAIN (N,NSEG,XFIXD,XBASE,DEL21,XPERT)
C
C.....DETERMINE PERTURBATION SOLUTION.
C
      DO 60 I=1,N
   60 YPERT(I)=YBASE(I)+DEL2*YUNIT(I)
C
C.....ADJUST VALUES NEAR CRITICAL POINT FOR MONOTONE BEHAVIOR.
C
      IF (LCORR .EQ. 1) CALL MONO (NCRD,LCRD,XPERT,YPERT)
C
C.....LOCATE MINIMUM, MAXIMUM, AND CRITICAL POINTS IN PERTURBATION
C     SOLUTION.
C
      CALL SCALE (N,XPERT,2,A,B)
      CALL SCALE (N,XPERT,1,A,B)
      CALL LOCATE (N,XPERT,YPERT,YCR2,IGRAD,LMN2,LMX2,NCR2,LCR2,XLOC2)
      WRITE (6,2130) ICASE,NCASE,M2,Q2,NAME,YCR2
      WRITE (6,2030)
      WRITE (6,2035)
      CALL UPLOW (A,B,XLOC2,6,NCR2+2,XOUT,FLAG)
      WRITE (6,2040) HEAD2,XOUT(1),FLAG(1),LMN2,XOUT(2),FLAG(2),LMX2
      IF (NCR2 .GT. 0) WRITE (6,2045) NCR2,
     Z      (ORD(I),XOUT(I+2),FLAG(I+2),LCR2(I),I=1,NCR2)
      CALL SCALE (N,XBASE,2,A,B)
C
C.....IF LCHEK .NE. 0 READ IN DATA FOR COMPARISON SOLUTION AND LOCATE
C     MINIMUM, MAXIMUM, AND CRITICAL POINTS.
C
      IF (LCHEK .EQ. 0) GO TO 90
      CALL INPUT(3)
      CALL SCALE (N,XCHEK,1,A,B)
      CALL LOCATE (N,XCHEK,YCHEK,YCR3,IGRAD,LMN3,LMX3,NCR3,LCR3,XLOC3)
      CALL UPLOW (A,B,XLOC3,6,NCR3+2,XOUT,FLAG)
      WRITE (6,2040) HEAD3,XOUT(1),FLAG(1),LMN3,XOUT(2),FLAG(2),LMX3
      IF (NCR3 .GT. 0) WRITE (6,2045) NCR3,
     Z      (ORD(I),XOUT(I+2),FLAG(I+2),LCR3(I),I=1,NCR3)
      CALL INTERP (N,XPERT,YPERT,XCHEK,YPRTI)
      CALL SCALE (N,XPERT,2,A,B)
      CALL SCALE (N,XCHEK,2,A,B)
      WRITE (6,2135) NAME,NAME,NAME,NAME,NAME
      WRITE (6,2140) (I,XBASE(I),YBASE(I),XCALB(I),YCALB(I),
     Z                XPERT(I),YPERT(I),XCHEK(I),YCHEK(I),YPRTI(I),
     Z                I=1,N)
      GO TO 100
   90 CONTINUE
C
      CALL SCALE (N,XPERT,2,A,B)
      WRITE (6,2145) NAME,NAME,NAME
      WRITE (6,2150) (I,XBASE(I),YBASE(I),XCALB(I),YCALB(I),
     Z                XPERT(I),YPERT(I),I=1,N)
  100 CONTINUE
C
C.....IF LCHEK .NE. 0 PLOT PERTURBATION AND COMPARISON SOLUTIONS.
C
      IF (LCHEK .EQ. 1) CALL PLOT (N,LPERT)
      CALL SCALE (N,XBASE,1,A,B)
  200 CONTINUE
      GO TO 999
C
C.....ABNORMAL TERMINATION OF COMPUTATION.
C
  900 WRITE (6,9000)
      GO TO 999
  905 WRITE (6,9050)
      GO TO 999
  910 WRITE (6,9100)
C
  999 WRITE (6,9500)
      STOP
```

```
C
C.....I/O FORMAT STATEMENTS FOLLOW.
C
 2000 FORMAT (1H1,132(1H*)/
     X        1X,1H*,25X,20A4,25X,1H*/
     X        1X,132(1H*)///)
 2010 FORMAT (1H ,10(1H<),1X,24HLIST OF INPUT PARAMETERS,1X,10(1H>)//
     X        6X,3HN =,1X,I3//
     X        6X,3HA =,1X,F4.1,4X,3HB =,1X,F4.1//
     X        6X,5A4,1X,4HMO =,1X,F6.4,4X,4HQO =,1X,F6.4//
     X        6X,5A4,1X,4HM1 =,1X,F6.4,4X,4HQ1 =,1X,F6.4///)
 2020 FORMAT (1H ,10(1H<),1X,18HCRITICAL VALUES OF,1X,A2,1X,10(1H>)//
     X        2(6X,5A4,1X,A2,6HCRIT =,1X,F7.4//)/)
 2030 FORMAT (1H ,5(1H<),1X,37HLOCATIONS OF MIN., MAX., AND CRITICAL,
     X                   1X,4HPTS.,1X,5(1H>))
 2035 FORMAT (1H ,2X,34H(* DENOTES POINT ON LOWER SURFACE))
 2040 FORMAT (/6X,5A4//
     X        11X,14HMINIMUM AT X =,1X,F6.4,A1,3X,8H(POINT 0,I3,1H)/
     X        11X,14HMAXIMUM AT X =,1X,F6.4,A1,3X,8H(POINT 0,I3,1H))
 2045 FORMAT (1H ,10X,I1,1X,18HCRITICAL POINT(S):/
     X        (15X,A4,6HAT X =,1X,F6.4,A1,3X,
     X            14H(AFTER POINT 0,I3,1H)))
 2046 FORMAT (1H ,10X,2(5A4)/)
 2047 FORMAT (1H ,14X,2HX(,I3,1H),15X,2HX(,I3,1H))
 2050 FORMAT (///1X,10(1H<),1X,25HSTRAINING POINTS SELECTED,1X,
     X            10(1H>),//
     X        6X,24HNUMBER OF FIXED POINTS =,1X,I1//
     X        6X,51HFIXED POINTS SELECTED (IN ADDITION TO END POINTS) =
     X                      /)
 2060 FORMAT (1H ,10X,16HPOINT OF MINIMUM,1X,A2)
 2070 FORMAT (1H ,10X,16HPOINT OF MAXIMUM,1X,A2)
 2080 FORMAT (1H ,10X,A2,6HCRIT (,A4,3HOF ,I1,1H))
 2090 FORMAT (///1X,10(1H<),1X,24HLOCATION OF FIXED POINTS,1X,10(1H>))
 2100 FORMAT (/6X,5A4//
     X        (1H ,10X,5HXFIX(,I1,3H) =,1X,F6.4,A1))
 2110 FORMAT (1H1,27(1H*)/
     X        1X,1H*,1X,20HUNIT PERTURBATION OF,1X,A2,1X,1H*/
     X        1X,1H*,12X,1HC,12X,1H*/
     X        1X,1H*,1X,23HUNIT STRAINING OF XBASE,1X,1H*/
     X        1X,27(1H*)///
     X        1X,5HPOINT,4X,5HXBASE,4X,8HXSTRUNIT,3X,A2,4HUNIT/)
 2120 FORMAT (1H ,1X,I3,1X,3F10.4)
 2130 FORMAT (1H1,27(1H*)/
     X        1X,19H* OUTPUT FOR CASE 0,I1,4H OF ,I1,2H */
     X        1X,27(1H*)//
     X        6X,4HMO2 =,1X,F6.4//
     X        6X,4HQ2 =,1X,F6.4//
     X        6X,A2,6HCRIT =,1X,F7.4///)
 2135 FORMAT (///1X,5HPOINT,4X,5HXBASE,5X,A2,4HBASE,
     X                    4X,5HXCALB,5X,A2,4HCALB,
     X                    4X,5HXPERT,5X,A2,4HPERT,
     X                    4X,5HXCHEK,5X,A2,4HCHEK,
     X        2X,A2,9HPERT(INT)/)
 2140 FORMAT (1H ,1X,I3,1X,9F10.4)
 2145 FORMAT (///1X,5HPOINT,4X,5HXBASE,5X,A2,4HBASE,
     X                    4X,5HXCALB,5X,A2,4HCALB,
     X                    4X,5HXPERT,5X,A2,4HPERT/)
 2150 FORMAT (1H ,1X,I3,1X,6F10.4)
 9000 FORMAT (///1X,28HNUMBER OF CRITICAL POINTS IN/
     X             1X,30HBASE AND CALIBRATION SOLUTIONS/
     X             1X,31HARE UNEQUAL - CALCULATION ENDED)
 9050 FORMAT (///1X,25HNUMBER OF CRITICAL POINTS/
     X             1X,23HSELECTED EXCEEDS NUMBER/
     X             1X,30HACTUALLY LOCATED - CALCULATION/
     X             1X,5HENDED)
 9100 FORMAT (///1X,28HORDER OF SPECIFIED POINTS IN/
     X             1X,30HBASE AND CALIBRATION SOLUTIONS/
     X             1X,39HDOES NOT CORRESPOND - CALCULATION ENDED)
 9800 FORMAT (1H1)
      END
      SUBROUTINE INPUT (ICALL)
C
C     ALL INPUT FOR PROGRAM PERTURB IS READ BY THIS SUBROUTINE, AND IS
C     REQUIRED IN THE FOLLOWING ORDER (FOR DETAILS, REFER TO
C     ACCOMPANYING MANUAL).
C
C**** CARD 01 (16I5) ************************************************
C
C          N         NUMBER OF DATA POINTS IN BASE AND CALIBRATION
C                    SOLUTIONS.
C
C          NCASE     NUMBER OF CASES FOR WHICH PERTURBATION SOLUTIONS ARE
C                    TO BE COMPUTED.
C
C          LSPEC     CONTROLS HOW INVARIANT POINTS IN STRAINING ARE
C                    SPECIFIED (SEE CARD 02).
C
C                    LSPEC = 0 ... INVARIANT POINTS SELECTED FROM AMONG
C                                  THOSE LOCATED BY THE PROGRAM
C                    LSPEC = 1 ... INVARIANT POINTS PRESELECTED BY USER
C
C          LECHO     CONTROLS WHETHER OR NOT INPUT DECK IS PRINTED.
C
C                    LECHO = 0 ... NO OUTPUT
C                    LECHO = 1 ... OUTPUT
C
C          LUNIT     CONTROLS WHETHER OR NOT UNIT COORDINATE STRAINING
C                    AND UNIT PERTURBATION ARE PRINTED.
C
C                    LUNIT = 0 ... NO OUTPUT
C                    LUNIT = 1 ... OUTPUT
C
C          LCHEK     SPECIFIES WHETHER OR NOT PERTURBATION SOLUTION IS TO
C                    BE CHECKED WITH AND PLOTTED AGAINST AN EXACT
C                    COMPARISON SOLUTION.
C
C                    LCHEK = 0 ... NO COMPARISON, NO PLOT
C                    LCHEK = 1 ... COMPARISON, PLOT
C
C          LPERT     SPECIFIES TYPE OF PERTURBATION. OPERATIONAL ONLY
C                    WHEN LCHEK = 1 AND AFFECTS ONLY OUTPUT FROM PLOT
C                    SUBROUTINE.
C
C                    LPERT = 1 ... THICKNESS-RATIO PERTURBATION
C                    LPERT = 2 ... ANGLE-OF-ATTACK PERTURBATION
C                    LPERT = 3 ... MACH-NUMBER PERTURBATION
C
C**** CARD 02 (16I5) ************************************************
C
C     REQUIRED INPUT DEPENDS ON VALUE OF LSPEC:
```

```
C                                                                  
C     ZZZZZZZZZZZZZZZZZZZZZZZZZZ INPUT FOR LSPEC = 0 ZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZ
C     Z
C     Z   NSELCT    NUMBER OF POINTS (IN ADDITION TO END POINTS) TO BE
C     Z             HELD INVARIANT IN STRAINING. NOTE: 1 <= NSELCT <= 6.
C     Z
C     Z   LSELCT(I) ...
C     Z             ARRAY OF LENGTH 6 OF WHICH NSELCT ELEMENTS ARE READ
C     Z             IN. SPECIFIES NATURE OF POINTS TO BE HELD INVARIANT
C     Z             ACCORDING TO THE CODE:
C     Z
C     Z                     1 ... MINIMUM PT. HELD INVARIANT
C     Z                     2 ... MAXIMUM PT. HELD INVARIANT
C     Z                     3 ... 1ST CRITICAL PT. HELD INVARIANT
C     Z                     4 ... 2ND CRITICAL PT. HELD INVARIANT
C     Z                     5 ... 3RD CRITICAL PT. HELD INVARIANT
C     Z                     6 ... 4TH CRITICAL PT. HELD INVARIANT
C     Z
C     Z             NOTE THAT THE CODE NUMBERS CAN BE ASSIGNED IN ANY
C     Z             ORDER, E.G.
C     Z
C     Z                               LSELCT(1) = 1
C     Z                               LSELCT(2) = 3
C     Z                               LSELCT(3) = 4
C     Z
C     Z             IS EQUIVALENT TO
C     Z
C     Z                               LSELCT(1) = 4
C     Z                               LSELCT(2) = 1
C     Z                               LSELCT(3) = 3
C     Z
C     Z             BOTH CORRESPONDING TO NSELCT = 3 WITH THE MINIMUM,
C     Z             AND FIRST AND SECOND CRITICAL POINTS HELD INVARIANT.
C     Z
C     ZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZ
C
C     ZZZZZZZZZZZZZZZZZZZZZZZZZZ INPUT FOR LSPEC = 1 ZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZ
C     Z
C     Z   NSELCT    NUMBER OF POINTS (IN ADDITION TO END POINTS) TO BE
C     Z             HELD INVARIANT IN STRAINING. NOTE: 1 <= NSELCT <= 6.
C     Z
C     Z   LOC0(I)   ARRAY OF LENGTH 6 OF WHICH NSELCT ELEMENTS ARE READ
C     Z             IN. SPECIFIES SUBSRIPTS OF THOSE BASE FLOW POINTS
C     Z             WHICH ARE TO BE HELD INVARIANT.
C     Z
C     Z   LOC1(I)   ARRAY OF LENGTH 6 OF WHICH NSELCT ELEMENTS ARE READ
C     Z             IN. SPECIFIES SUBSCRIPTS OF THOSE CALIBRATION FLOW
C     Z             POINTS WHICH ARE TO BE HELD INVARIANT.
C     Z
C     ZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZZ
C
C**** CARD 03 (20A4) *******************************************************
C
C         TI"LE     IDENTIFIES JOB - PRINTED AS HEADLINE ON FIRST PAGE
C                   OF OUTPUT.
C
C**** CARD 04 (A2) *********************************************************
C
C         NAME      CHARACTER STRING OF LENGTH 2 WHICH SYMBOLIZES
C                   DEPENDENT VARIABLE, E.G. 'CP' FOR PRESSURE
```

```
C                   COEFFICIENT.
C
C**** CARD 05 (8F10.6) *****************************************************
C
C         A         SCALING PARAMETER (A = -X(1)), WHERE X(1) IS FIRST
C                   DATA POINT ON LOWER SURFACE ... SEE MANUAL).
C
C         B         SCALING PARAMETER (B = X(N)), WHERE X(N) IS LAST DATA
C                   POINT ON UPPER SURFACE ... SEE MANUAL).
C
C**** CARD 06 (8F10.6) *****************************************************
C
C         M0        ONCOMING MACH NUMBER IN BASE SOLUTION.
C
C         Q0        VALUE OF PERTURBATION PARAMETER IN BASE SOLUTION.
C
C**** ONE SET OF K CARDS (8F10.6), WHERE K = 1 + INT(N/8) ******************
C
C         XBASE(I), I=1,N ...
C                   X COORDINATE IN BAS: SOLUTION.
C
C**** ONE SET OF K CARDS (8F10.6), K AS ABOVE ******************************
C
C         YBASE(I), I=1,N ...
C                   DEPENDENT VARIABLE IN BASE SOLUTION.
C
C**** NEXT CARD (8F10.6) ***************************************************
C
C         M1        ONCOMING MACH NUMBER IN CALIBRATION SOLUTION.
C
C         Q1        VALUE OF PERTURBATION PARAMETER IN CALIBRATION
C                   SOLUTION.
C
C**** ONE SET OF K CARDS (8F10.6), K AS ABOVE ******************************
C
C         XCALB(I), I= 1,N ...
C                   X COORDINATE IN CALIBRATION SOLUTION.
C
C**** ONE SET OF K CARDS (8F10.6), K AS ABOVE ******************************
C
C         YCALB(I), I=1,N ...
C                   DEPENDENT VARIABLE IN CALIBRATION SOLUTION..
C
C**** :CASE SETS OF CARDS, EACH SET COMPRISED AS FOLLOWS: ******************
C
C              **** FIRST CARD (8F10.6) ************************************
C
C               M2        ONCOMING MACH NUMBER IN SOLUTION TO BE
C                         COMPUTED.
C
C               Q2        VALUE OF PERTURBATION PARAMETER IN SOLUTION TO
C                         BE COMPUTED.
C
C              **** ONE SET OF K CARDS (8F10.6), K AS ABOVE ***************
C
C               XCHEK(I), I= 1,N ...
C                         X COORDINATE IN COMPARISON SOLUTION.
C
C              **** ONE SET OF K CARDS (8F10.6), K AS ABOVE ***************
C
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C              YCHEK(I), I=1,N ...
C                        DEPENDENT VARIABLE IN COMPARISON SOLUTION.
C
C     ********    THE LATTER TWO SETS OF K CARDS ARE
C     * NOTE *    OMITTED WHEN LCHEK = 0 (NO COMPARISON
C     ********    SOLUTION AVAILABLE).
C
      DIMENSION LOCO(6),LOC1(6),LSELCT(6),TITLE(20)
      DIMENSION XBASE(200),XCALB(200),XPERT(200),XCHEK(200),
     Z          YBASE(200),YCALB(200),YPERT(200),YCHEK(200)
      REAL M0,M1,M2
      INTEGER*2 NAME
      COMMON /PARAM/ TITLE,LOCO,LOC1,LSELCT,N,NCASE,LSPEC,LECHO,LUNIT,
     Z               LCHEK,LPERT,NSELCT,A,B,NAME
      COMMON /PERT/ M0,M1,M2,Q0,Q1,Q2,YCR0,YCR1,YCR2
      COMMON /XY/ XBASE,XCALB,XPERT,XCHEK,YBASE,YCALB,YPERT,YCHEK
      GO TO (100,200,300), ICALL
  100 READ (5,1000) N,NCASE,LSPEC,LECHO,LUNIT,LCHEK,LPERT
      IF (LSPEC .EQ. 0) READ (5,1000) NSELCT,(LSELCT(I),I=1,NSELCT)
      IF (LSPEC .EQ. 1) READ (5,1000) NSELCT,(LOCO(I),I=1,NSELCT),
     Z                                 (LOC1(I),I=1,NSELCT)
      READ (5,1050) TITLE
      READ (5,1100) NAME
      READ (5,1200) A,B
      READ (5,1200) M0,Q0
      READ (5,1200) (XBASE(I),I=1,N)
      READ (5,1200) (YBASE(I),I=1,N)
      READ (5,1200) M1,Q1
      READ (5,1200) (XCALB(I),I=1,N)
      READ (5,1200) (YCALB(I),I=1,N)
      RETURN
  200 READ (5,1200) M2,Q2
      RETURN
  300 READ (5,1200) (XCHEK(I),I=1,N)
      READ (5,1200) (YCHEK(I),I=1,N)
      RETURN
 1000 FORMAT (16I5)
 1050 FORMAT (20A4)
 1100 FORMAT (A2)
 1200 FORMAT (8F10.6)
      END
      SUBROUTINE ECHINP
      DIMENSION LOCO(6),LOC1(6),LSELCT(6),TITLE(20)
      DIMENSION XBASE(200),XCALB(200),XPERT(200),XCHEK(200),
     Z          YBASE(200),YCALB(200),YPERT(200),YCHEK(200)
      REAL M0,M1,M2
      INTEGER*2 NAME
      COMMON /PARAM/ TITLE,LOCO,LOC1,LSELCT,N,NCASE,LSPEC,LECHO,LUNIT,
     Z               LCHEK,LPERT,NSELCT,A,B,NAME
      COMMON /PERT/ M0,M1,M2,Q0,Q1,Q2,YCR0,YCR1,YCR2
      COMMON /XY/ XBASE,XCALB,XPERT,XCHEK,YBASE,YCALB,YPERT,YCHEK
      WRITE (6,1400)
      WRITE (6,1500) N,NCASE,LSPEC,LECHO,LUNIT,LCHEK,LPERT
      IF (LSPEC .EQ. 0) WRITE (6,1500) NSELCT,(LSELCT(I),I=1,NSELCT)
      IF (LSPEC .EQ. 1) WRITE (6,1500) NSELCT,(LOCO(I),I=1,NSELCT),
     Z                                  (LOC1(I),I=1,NSELCT)
      WRITE (6,1550) TITLE
      WRITE (6,1600) NAME
      WRITE (6,1700) A,B
      WRITE (6,1700) M0,Q0
      WRITE (6,1700) (XBASE(I),I=1,N)
      WRITE (6,1700) (YBASE(I),I=1,N)
      WRITE (6,1700) M1,Q1
      WRITE (6,1700) (XCALB(I),I=1,N)
      WRITE (6,1700) (YCALB(I),I=1,N)
      RETURN
 1400 FORMAT (1H1,25(1H*)/
     Z        1X,1H*,1X,21HLISTING OF INPUT DECK,1X,1H*/
     Z        1X,25(1H*)///)
 1500 FORMAT (1X,16I5)
 1550 FORMAT (1X,20A4)
 1600 FORMAT (1X,A2)
 1700 FORMAT (1X,8F10.6)
      END
      SUBROUTINE BANNER
      WRITE (6,1300)
      WRITE (6,1310)
 1300 FORMAT (1H1,10(/),49X,55(1H*)/49X,1H*,53X,1H*/
     Z 49X,1H*,19X,15HPROGRAM PERTURB,19X,1H*/49X,1H*,53X,1H*/
     Z 49X,1H*,8X,37HCALCULATES NONLINEAR SINGLE-PARAMETER,
     Z                                    8X,1H*/49X,1H*,53X,1H*/
     Z 49X,1H*,13X,27HCONTINUOUS OR DISCONTINUOUS,
     Z                                    13X,1H*/49X,1H*,53X,1H*/
     Z 49X,1H*,15X,22HPERTURBATION SOLUTIONS,16X,1H*/49X,1H*,53X,1H*/
     Z 49X,1H*,9X,34HWHICH REPRESENT A CHANGE IN EITHER,
     Z                                    10X,1H*/49X,1H*,53X,1H*/
     Z 49X,1H*,13X,27HGEOMETRY OR FLOW CONDITIONS,
     Z                                    13X,1H*/49X,1H*,53X,1H*/
     Z 49X,1H*,4X,44HBY EMPLOYING A STRAINED-COORDINATE PROCEDURE,
     Z                                    5X,1H*/49X,1H*,53X,1H*/
     Z 49X,1H*,4X,45HUTILIZING A UNIT PERTURBATION DETERMINED FROM,
     Z                                    4X,1H*/49X,1H*,53X,1H*/
     Z 49X,1H*,14X,25HTWO PREVIOUSLY CALCULATED,
     Z                                    14X,1H*/49X,1H*,53X,1H*)
 1310 FORMAT ( 49X,1H*,9X,34H'BASE' AND 'CALIBRATION' SOLUTIONS
     Z                                    10X,1H*/49X,1H*,53X,1H*/
     Z 49X,1H*,4X,45HDISPLACED FROM ONE ANOTHER BY SOME REASONABLE,
     Z                                    4X,1H*/49X,1H*,53X,1H*/
     Z 49X,1H*,8X,36HCHANGE IN GEOMETRY OR FLOW CONDITION,
     Z                                    9X,1H*/49X,1H*,53X,1H*/
     Z                                          49X,1H*,53X,1H*/
     Z 49X,1H*,21X,10HWRITTEN BY,22X,1H*/49X,1H*,53X,1H*/
     Z 49X,1H*,7X,39HJAMES P. ELLIOTT AND STEPHEN S. STAHARA,
     Z                                    7X,1H*/49X,1H*,53X,1H*/
     Z                                          49X,1H*,53X,1H*/
     Z 49X,1H*,7X,38HNIELSEN ENGINEERING AND RESEARCH, INC.,
     Z                                    8X,1H*/49X,1H*,53X,1H*/
     Z 49X,1H*,14X,25HMOUNTAIN VIEW, CALIFORNIA,14X,1H*/49X,1H*,53X,1H*/
     Z                                   49X,55(1H*))
      RETURN
      END
      SUBROUTINE SCALE (N,X,M,A,B)
C
C.....ENTRY WITH M = 1 CONVERTS FROM PHYSICAL X (0 TO -A ON LOWER
C     SURFACE, 0 TO B ON UPPER SURFACE) TO NORMALIZED X (0 < X < 1).
C     ENTRY WITH M=2 REVERSES THE PROCESS. NZ (DETERMINED WHEN M=1)
C     CORRESPONDS TO POINT AT NOSE OF BLADE OR AIRFOIL.
C
      COMMON /FLOREV/ NZ
      DIMENSION X(200)
```

```
      GO TO (1,6),M
    1 CONTINUE
      NZ=0
      DO 2 I=2,N
      IF (X(I) .LT. X(I-1)) NZ=I
    2 CONTINUE
      DO 5 I=1,N
      IF (I .LE. NZ) T=-X(I)
      IF (I .GT. NZ) T=X(I)
      X(I)=(T-A)/(B-A)
    5 CONTINUE
      RETURN
    6 DO 7 I=1,N
      X(I)=ABS((B-A)*X(I)+A)
    7 CONTINUE
      RETURN
      END
      SUBROUTINE LOCATE (N,X,Y,YCRIT,IGRAD,LMIN,LMAX,NCRIT,LCRIT,XLOC)
C
C.....OPERATES ON THE INPUT ARRAY Y, LOCATING MINIMUM AND MAXIMUM
C     VALUES, AND ALL CRITICAL POINTS (Y=YCRIT) FOR WHICH DY/DX (IN
C     PHYSICAL COORDINATES) HAS ALGEBRAIC SIGN GIVEN BY IGRAD. NCRIT
C     IS NUMBER OF CRITICAL POINTS. POINTS FOUND ARE STORED IN THE ARRAY
C     XLOC AS FOLLOWS:
C
C                             XLOC(1) = MINIMUM PT.
C                             XLOC(2) = MAXIMUM PT.
C                             XLOC(3) = CRITICAL PT. 01
C                             ...     = ...
C                             XLOC(6) = CRITICAL PT. 04
C
      DIMENSION X(200),Y(200),LCRIT(4),XCRIT(4),XLOC(6)
      COMMON /FLOREV/ IREV
      IFLOW=-1
      LMIN=1
      LMAX=1
      ISTART=2
      IF (IREV .EQ. 0) GO TO 5
      LMIN=2
      LMAX=2
      ISTART=3
    5 CONTINUE
      NCRIT=0
      DO 100 I=ISTART,N
      IF (IREV .NE. 0 .AND. I .EQ. N) GO TO 10
      IF (Y(I) .GT. Y(LMAX)) LMAX=I
      IF (Y(I) .LT. Y(LMIN)) LMIN=I
   10 CONTINUE
      IF ((Y(I) .GT. YCRIT .AND. Y(I-1) .GT. YCRIT) .OR.
     X    (Y(I) .LT. YCRIT .AND. Y(I-1) .LT. YCRIT)) GO TO 100
      IF (I .GT. IREV) IFLOW=1
      IF (((Y(I)-Y(I-1))*FLOAT(IFLOW*IGRAD) .LT. 0.0) GO TO 100
      NCRIT=NCRIT+1
      LCRIT(NCRIT)=I-1
      SLOPE=(X(I)-X(I-1))/(Y(I)-Y(I-1))
      XCRIT(NCRIT)=X(I-1)+SLOPE*(YCRIT-Y(I-1))
  100 CONTINUE
      XLOC(1)=X(LMIN)
      XLOC(2)=X(LMAX)
      IF (NCRIT .EQ. 0) RETURN
```

```
      DO 200 I=1,NCRIT
  200 XLOC(I+2)=XCRIT(I)
      RETURN
      END
      SUBROUTINE SORT (N,X,ISEQ)
C
C.....ARRANGES THE SET X(1), X(2), ... , X(N) IN A MONOTONE INCREASING
C     SEQUENCE. ISEQ GIVES ORDER OF SUBSCRIPTS IN REARRANGED SEQUENCE.
C
      DIMENSION X(8),ISEQ(8)
      NM1=N-1
      DO 1 I=1,N
    1 ISEQ(I)=I
   10 ITEST=0
      DO 100 I=1,NM1
      IF (X(I) .LE. X(I+1)) GO TO 100
      XSAVE=X(I)
      X(I)=X(I+1)
      X(I+1)=XSAVE
      ISAVE=ISEQ(I)
      ISEQ(I)=ISEQ(I+1)
      ISEQ(I+1)=ISAVE
      ITEST=1
  100 CONTINUE
      IF (ITEST .EQ. 1) GO TO 10
      RETURN
      END
      SUBROUTINE INTERP (N,X,Y,XI,YI)
C
C.....GIVEN THE SET OF POINTS X(I), Y(I), I=1,N, AND THE SET XI(J),
C     J=1,N, USES LINEAR INTERPOLATION TO COMPUTE THE SET YI(J), J=1,N.
C
      DIMENSION X(200),Y(200),XI(200),YI(200)
      NM1=N-1
      JSTART=1
      DO 100 I=1,N
      IF (XI(I) .LE. X(1)) GO TO 10
      IF (XI(I) .GE. X(N)) GO TO 20
      GO TO 30
   10 J=1
      GO TO 95
   20 J=N-1
      GO TO 95
   30 CONTINUE
      DO 90 J=JSTART,NM1
      IF (XI(I) .NE. X(J)) GO TO 40
      YI(I)=Y(J)
      GO TO 100
   40 IF (XI(I) .GT. X(J) .AND. XI(I) .LT. X(J+1)) GO TO 95
   90 CONTINUE
   95 SLOPE=(Y(J+1)-Y(J))/(X(J+1)-X(J))
      YI(I)=Y(J)+SLOPE*(XI(I)-X(J))
      JSTART=J
  100 CONTINUE
      RETURN
      END
      SUBROUTINE STRAIN (N,NSEG,XFIX,XIN,PARM,XOUT)
C
C.....COMPUTES STRAINED COORDINATE FROM INPUT ARRAY XIN, USING PIECEWISE
C     LINEAR STRAINING WITH NSEG LINEAR SEGMENTS. FOR UNIT STRAINING,
```

```
C     INPUT VALUE OF PARM IS 1.0; FOR GENERAL CASE.
C                            PARM = (Q2-Q0)/(Q1-Q0).
C
      DIMENSION XFIX(8),XIN(200),XOUT(200)
      COMMON /COEFF/ C(7),D(7)
      JSTART=1
      DO 50 I=1,N
      DO 40 J=JSTART,NSEG
      IF (XIN(I) .GE. XFIX(J) .AND. XIN(I) .LE. XFIX(J+1)) GO TO 45
   40 CONTINUE
   45 XOUT(I)=XIN(I)+PARM*(C(J)+(D(J)-1.0)*XIN(I))
      JSTART=J
   50 CONTINUE
      RETURN
      END
      SUBROUTINE MONO (N,L,X,Y)
C
C.....CHECKS POINTS IN VICINITY OF A CRITICAL POINT FOR MONOTONE
C     BEHAVIOR, AND ADJUSTS VALUES TO GIVE A LINEAR PROFILE.
C
      DIMENSION L(4),X(200),Y(200)
      DO 100 I=1,N
      LS=L(I)
      Y1=Y(LS-1)
      Y2=Y(LS)
      Y3=Y(LS+1)
      Y4=Y(LS+2)
      IF ((Y1 .LT. Y2) .AND. (Y2 .LT. Y3) .AND. (Y3 .LT. Y4)) GO TO 100
      IF ((Y1 .GT. Y2) .AND. (Y2 .GT. Y3) .AND. (Y3 .GT. Y4)) GO TO 100
      X1=X(LS-1)
      X2=X(LS)
      X3=X(LS+1)
      X4=X(LS+2)
      SLOPE=(Y4-Y1)/(X4-X1)
      Y(LS)=Y1+SLOPE*(X2-X1)
      Y(LS+1)=Y1+SLOPE*(X3-X1)
  100 CONTINUE
      RETURN
      END
      SUBROUTINE PLOT (N,LPERT)
C
C.....CREATES FILE OF COMMANDS FOR PROGRAM 'TOPDRAW' AT STANFORD CENTER
C     FOR INFORMATION PROCESSING (S.C.I.P.). CALLED ONLY ONCE IN MAIN
C     PROGRAM AND MAY BE DELETED OR REPLACED.
C
      DIMENSION X0(200),X1(200),X2(200),X3(200),
     %          Y0(200),Y1(200),Y2(200),Y3(200)
      COMMON /PERT/ M0,M1,M2,Q0,Q1,Q2,YCR0,YCR1,YCR2
      COMMON /XY/ X0,X1,X2,X3,Y0,Y1,Y2,Y3
      LOGICAL*1 SYM(3) /1HT,1HA,1HM/, SUB(3) /1HG,1HG,1H /,
     %          S0 /1HB/, S1 /1HC/
      REAL M0,M1,M2
      DATA ICALL /0/, XL,XR0,XT0,XR1,XT1,XR2,XT2
     %          /0.0, 0.12, 0.14, 0.16, 0.18, 0.2, 0.24/
      IF (ICALL .EQ. 0) WRITE (4,1000)
      ICALL=1
      CALL LIMITS (N,YMIN,YMAX)
      WRITE (4,1050)
      WRITE (4,1100) YMIN,YMAX
      WRITE (4,1200)
      WRITE (4,1300)
      WRITE (4,1350) SYM(LPERT),SYM(LPERT),Q0,SYM(LPERT),Q1
      WRITE (4,1360)
      WRITE (4,1370) SUB(LPERT),SUB(LPERT),SUB(LPERT)
      WRITE (4,1400)
      IF (LPERT .NE. 3) WRITE (4,1500) M0,SYM(LPERT),Q2
      IF (LPERT .EQ. 3) WRITE (4,1505) Q2
      IF (LPERT .NE. 3) GO TO 10
      WRITE (4,5000) XL,YCR0,XR0,YCR0
      WRITE (4,1700)
      WRITE (4,1530) XT0,YCR0,S0
      WRITE (4,5000) XL,YCR1,XR1,YCR1
      WRITE (4,1800)
      WRITE (4,1530) XT1,YCR1,S1
   10 CONTINUE
      WRITE (4,5000) XL,YCR2,XR2,YCR2
      WRITE (4,1510)
      WRITE (4,1550) XT2,YCR2
      WRITE (4,5000) (X0(I),Y0(I),I=1,N)
      WRITE (4,1700)
      WRITE (4,5000) (X1(I),Y1(I),I=1,N)
      WRITE (4,1800)
      WRITE (4,5000) (X2(I),Y2(I),I=1,N)
      WRITE (4,1900)
      WRITE (4,5000) (X3(I),Y3(I),I=1,N)
      WRITE (4,1510)
      RETURN
 1000 FORMAT (45H//PERTPLOT JOB ,'     JIM ELLIOTT',REGION=512K/
     %          19H//DRAW EXEC TOPDRAW/
     %          40HSET DEVICE VERSATEC CONTINUOUS INTENSITY/
     %          18HSET CARD LENGTH 80/
     %          15HSET FONT DUPLEX)
 1050 FORMAT (9HNEW FRAME/
     %          27HSET TICKS TOP OFF RIGHT OFF/
     %          22HSET WINDOW X 2 7 Y 2 8/
     %          20HSET SYMBOL 8P SIZE 1)
 1100 FORMAT (18HSET LIMITS X 0 1 Y,2F5.1)
 1200 FORMAT (51HTITLE 4.5 9.5 CENTER SIZE 3 SPACES 7 'PLOT OF C2P3'/
     %          19HCASE ' LLL LL  CLC')
 1300 FORMAT (39HTITLE 4.5 8.7 CENTER SIZE 1.5 SPACES 38,1X,
     %          24H'FULL F2HOLST3   NONL. ')
 1350 FORMAT (4HMORE,1X,1H',A1,1X,5HPERT.,3X,A1,5H2B3 =,F6.3,3X,A1,
     %          5H2C3 =,F6.3,1H')
 1360 FORMAT (4HCASE,1X,24H' LLL GC LLLLC    LLL  ')
 1370 FORMAT (4HCASE,1X,1H',A1,2X,3HLLL,4X,A1,3HCLC,11X,A1,3HCLC,8X,1H')
 1400 FORMAT (25HTITLE 0.8 5 SIZE 2 'C2P3'/
     %          4HCASE,1X,6H' CLC'/
     %          23HTITLE BOTTOM SIZE 2 'X')
 1500 FORMAT (47HTITLE 4.5 0.5 CENTER SIZE 1.5 SPACES 17 'M2O3 =,
     %          F6.3,3X,A1,2H =,F6.3,1H'/
     %          10HCASE ' CSC,11X,1HG,8X,1H')
 1505 FORMAT (34HTITLE 4.5 0.5 CENTER SIZE 1.5 'M =,F6.3,1H')
 1510 FORMAT (6HJOIN 1)
 1530 FORMAT (5HTITLE,1X,2(F6.3,1X),
     %          27HDATA SIZE 1.0 '(C42P350*1)2,A1,2H3'/
     %          4HCASE,1X,16H'  CCLCCC C CLC')
 1550 FORMAT (5HTITLE,1X,2(F6.3,1X),25HDATA SIZE 1.5 'C42P350*1'/
     %          16HCASE ' CCLCCC C')
 1700 FORMAT (15HSET INTENSITY 3/
     %          11HJOIN 1 DOTS/
```

```
     X          15HSET INTENSITY 2)                                    PLOT0082
 1800 FORMAT (13HJOIN 1 DASHES)                                        PLOT0083
 1900 FORMAT (4HPLOT)                                                  PLOT0084
 5000 FORMAT (5(2(F6.3,1X),1H;))                                       PLOT0085
      END                                                              PLOT0086
      SUBROUTINE LIMITS (N,YMIN,YMAX)                                  LIMI0001
C                                                                      LIMI0002
C.....SEARCHES FOUR DATA ARRAYS Y0,Y1,Y2,Y3 FOR MINIMUM AND MAXIMUM.   LIMI0003
C     CALLED ONLY BY PLOT SUBROUTINE.                                  LIMI0004
C                                                                      LIMI0005
      DIMENSION X0(200),X1(200),X2(200),X3(200),                       LIMI0006
     X          Y0(200),Y1(200),Y2(200),Y3(200)                        LIMI0007
      DIMENSION Z(800)                                                 LIMI0008
      COMMON /XY/ X0,X1,X2,X3,Y0,Y1,Y2,Y3                              LIMI0009
      EQUIVALENCE (Y0(1),Z(1))                                         LIMI0010
      YMIN=Z(1)                                                        LIMI0011
      YMAX=Z(1)                                                        LIMI0012
      DO 10 I=1,4                                                      LIMI0013
      JSTART=200*(I-1)+1                                               LIMI0014
      JSTOP=JSTART+N-1                                                 LIMI0015
      DO 10 J=JSTART,JSTOP                                             LIMI0016
      IF (Z(J).GT.YMAX) YMAX=Z(J)                                      LIMI0017
      IF (Z(J).LT.YMIN) YMIN=Z(J)                                      LIMI0018
   10 CONTINUE                                                         LIMI0019
      YSAVE=YMAX                                                       LIMI0020
      YMAX=YMIN                                                        LIMI0021
      YMIN=YSAVE                                                       LIMI0022
      CALL ROUND (YMIN)                                                LIMI0023
      CALL ROUND (YMAX)                                                LIMI0024
      RETURN                                                           LIMI0025
      END                                                              LIMI0026
      SUBROUTINE ROUND (Y)                                             ROUN0001
C                                                                      ROUN0002
C.....ROUNDS Y LIMITS FOR OUTPUT IN F5.1 FORMAT. CALLED ONLY BY        ROUN0003
C     SUBROUTINE LIMITS.                                               ROUN0004
C                                                                      ROUN0005
      Z=ABS(Y)                                                         ROUN0006
      IF (10.*Z-INT(10.*Z).LT..5) Z=Z+.05                              ROUN0007
      IF (Y.GT.0.) GO TO 1                                             ROUN0008
      Y=-Z                                                             ROUN0009
      RETURN                                                           ROUN0010
    1 Y=Z                                                              ROUN0011
      RETURN                                                           ROUN0012
      END                                                              ROUN0013
      SUBROUTINE UPLOW (A,B,XIN,K,N,XOUT,FLAG)                         UPLO0001
C                                                                      UPLO0002
C.....CONVERTS NORMALIZED ARRAY XIN TO PHYSICAL ARRAY XOUT AND FLAGS   UPLO0003
C     POINTS ON LOWER SURFACE WITH A '*'.                              UPLO0004
C                                                                      UPLO0005
      DIMENSION XIN(K),XOUT(8)                                         UPLO0006
      LOGICAL*1 FLAG(8), BLANK/1H /, STAR/1H*/                         UPLO0007
      XNOSE=-A/(B-A)                                                   UPLO0008
      DO 1 I=1,N                                                       UPLO0009
      FLAG(I)=BLANK                                                    UPLO0010
      IF (XIN(I) .LT. XNOSE) FLAG(I)=STAR                              UPLO0011
      XOUT(I)=ABS(B-A)*XIN(I)+A                                        UPLO0012
    1 CONTINUE                                                         UPLO0013
      RETURN                                                           UPLO0014
      END                                                              UPLO0015
```

APPENDIX C

LIST OF SYMBOLS

C	blade chord, m
H	blade spacing for nonstaggered cascades, m
i	invariant point index; eq. (6); also, index for Lagrangian coefficients; eq. (22)
k	dummy index; eq. (20)
L	two-dimensional full potential operator; eq. (2)
L_1	linear operator representing first-order perturbation of two-dimensional full potential equation; eq. (4)
L_2	linear operator representing first-order perturbation terms arising from coordinate straining; eq. (9)
L_i	Lagrangian coefficients; eq. (22)
n	total number of shock points and high-gradient maxima points; eq. (24)
N	total number of invariant points, equal to n + 2; eq. (24)
q	arbitrary geometric or flow parameter to be perturbed; eq. (13)
q_c	calibration flow value of q; eq. (9)
q_o	base flow value of q; eq. (3)
Q	approximate flow solution for arbitrary flow quantity; eq. (1)
Q_c	calibration flow solution for value q_c of arbitrary parameter; eq. (8)
Q_o	base flow solution for value q_o of arbitrary parameter; eq. (1)
Q_p	linearized perturbation solution per unit change of perturbed parameter; eq. (1)
(s,t)	strained (x,y) coordinates; eq. (5)

(x,y)	nondimensional blade-fixed orthogonal coordinates; eq. (5), normalized by C
$(\bar{x},\bar{y})$	nondimensional blade-fixed orthogonal coordinates related to calibration solution; eq. (9)
(x_1,y_1)	straining functions associated with (x,y) coordinates; eq. (6)
(x_{1_i},y_{1_i})	straining functions associated with ith invariant point; eq. (6)
$(\delta x_i,\delta y_i)$	unit displacements in (x,y) directions associated with ith invariant point; eq. (6)
δx_i^c	unit displacement in x direction between base and calibration flows of the ith invariant point; eq. (18)
ϵ	perturbation change of geometric or flow parameter; eq. (17)
ϵ_c	perturbation of geometric or flow parameter between base and calibration flows; eq. (18)
Φ	nondimensional total velocity potential; eq. (2), normalized by CV_∞
Φ_o	nondimensional base flow velocity ptential; eq. (3), normalized by CV_∞
Φ_1	nondimensional perturbation velocity potential; eq. (3), normalized by CV_∞

Subscripts

o	denotes base flow quantities
1	denotes perturbation quantities
c	denotes quantities associated with calibration flow

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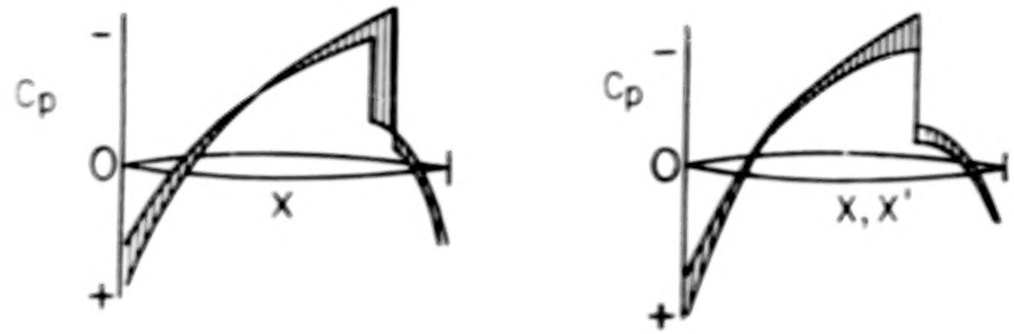
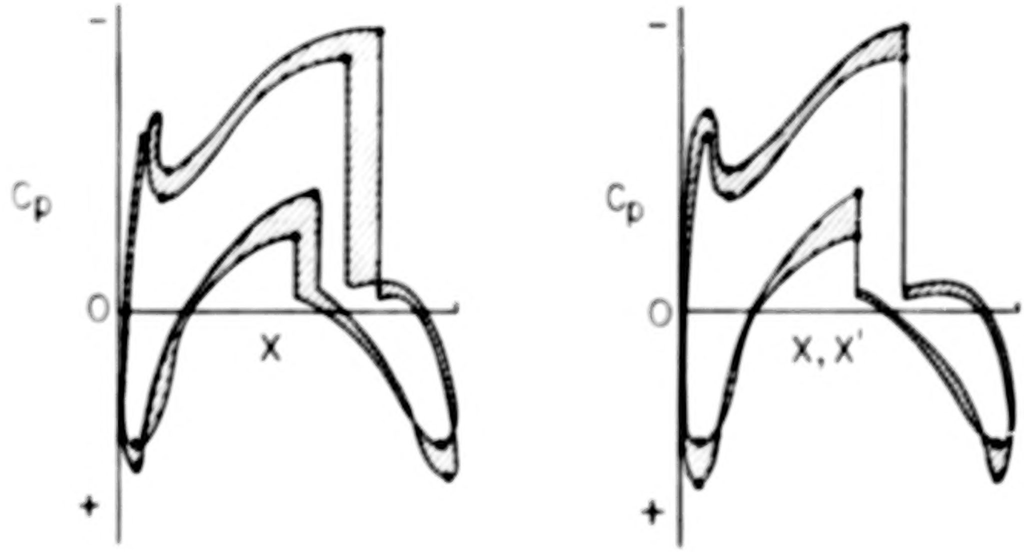
Perturbation for calibration solution in physical coordinates

Perturbation for calibration solution in strained coordinates



(a) Single shock.



(b) Multiple shock and high-gradient locations.

Figure 1.- Illustration of perturbation solution for calibration solution in physical and strained coordinates

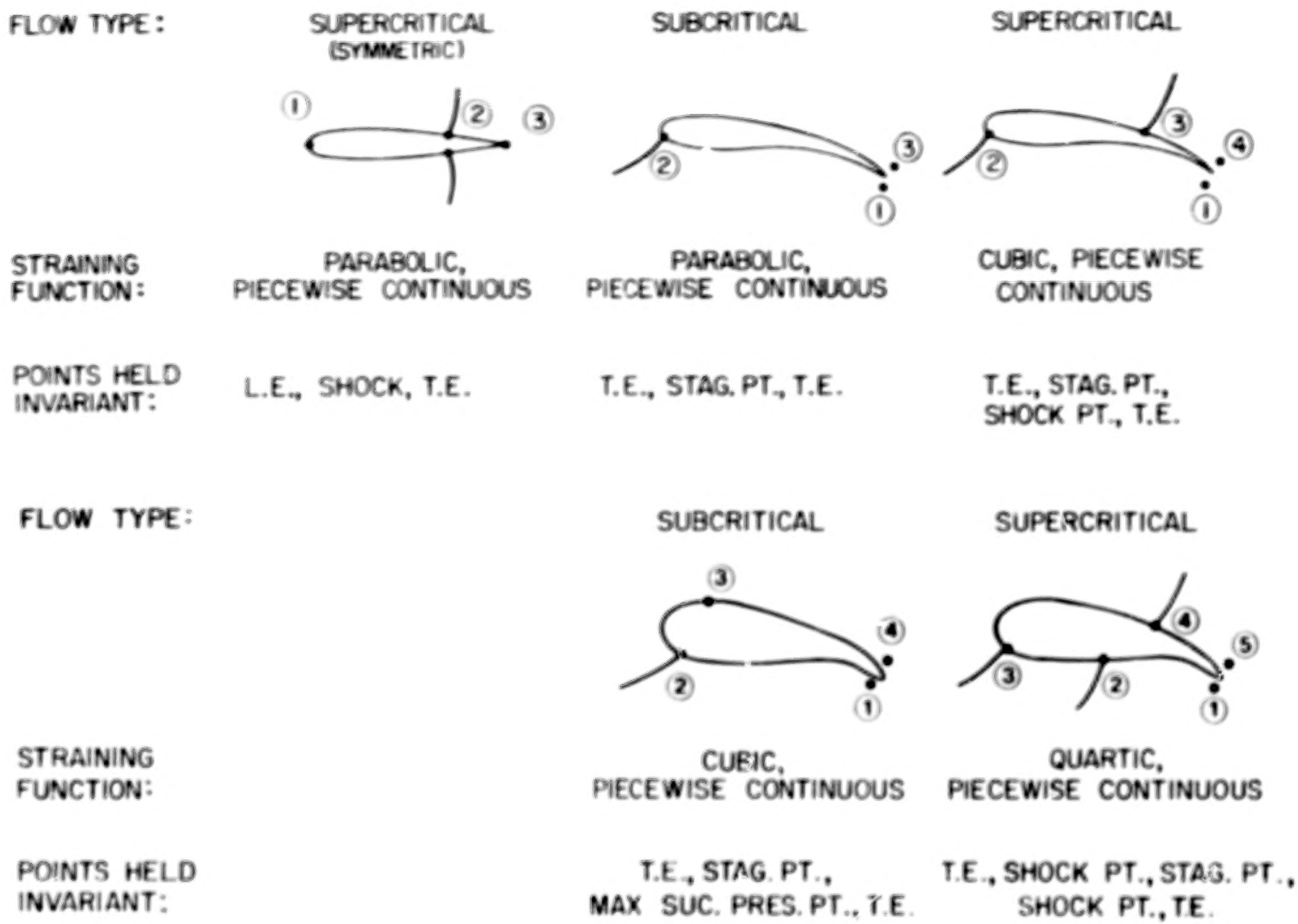


Figure 2.- Summary of various flows and straining functions considered

TABLE OF CONTENTS

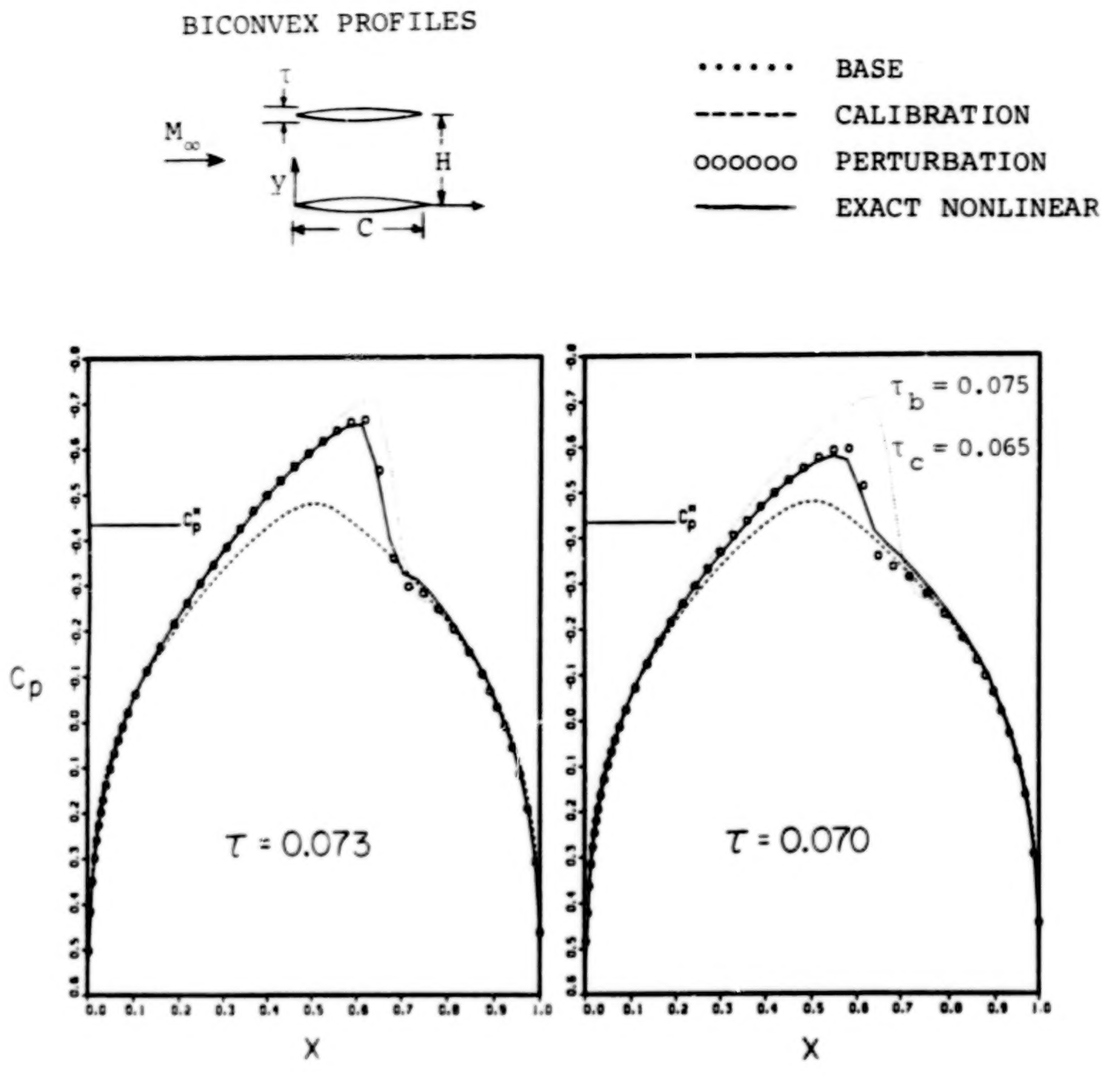


Figure 3.- Comparison of perturbation (O) and nonlinear (—) surface pressures for a thickness-ratio perturbation of a nonlifting cascade of biconvex profiles with H/C = 1.0 at M_∞ = 0.80

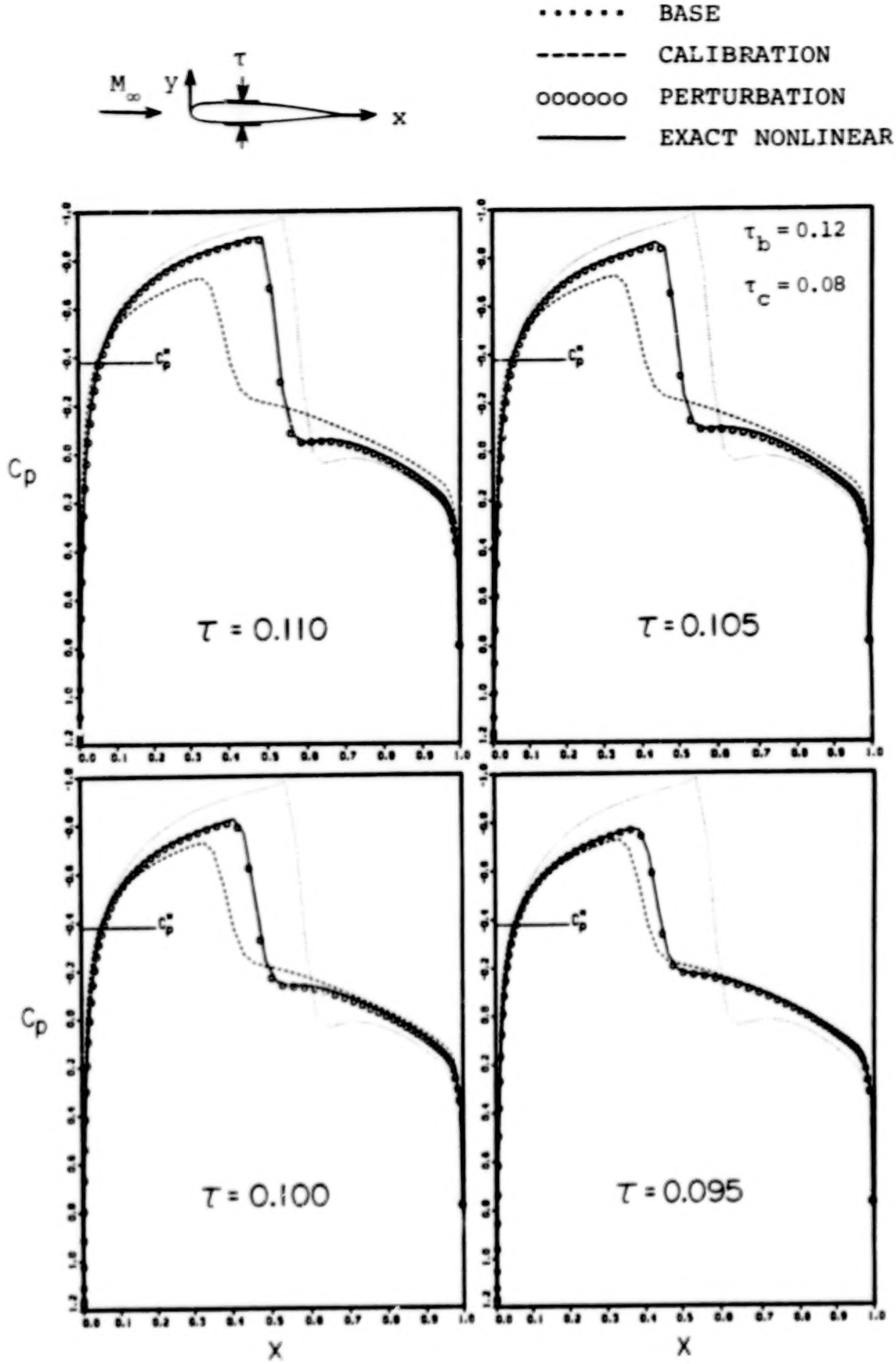


Figure 4.- Comparison of perturbation (O) and nonlinear (—) surface pressures for a thickness-ratio perturbation for an isolated NACA 00XX airfoil at $M_\infty = 0.820$ and $\alpha = 0°$ for solution interpolation

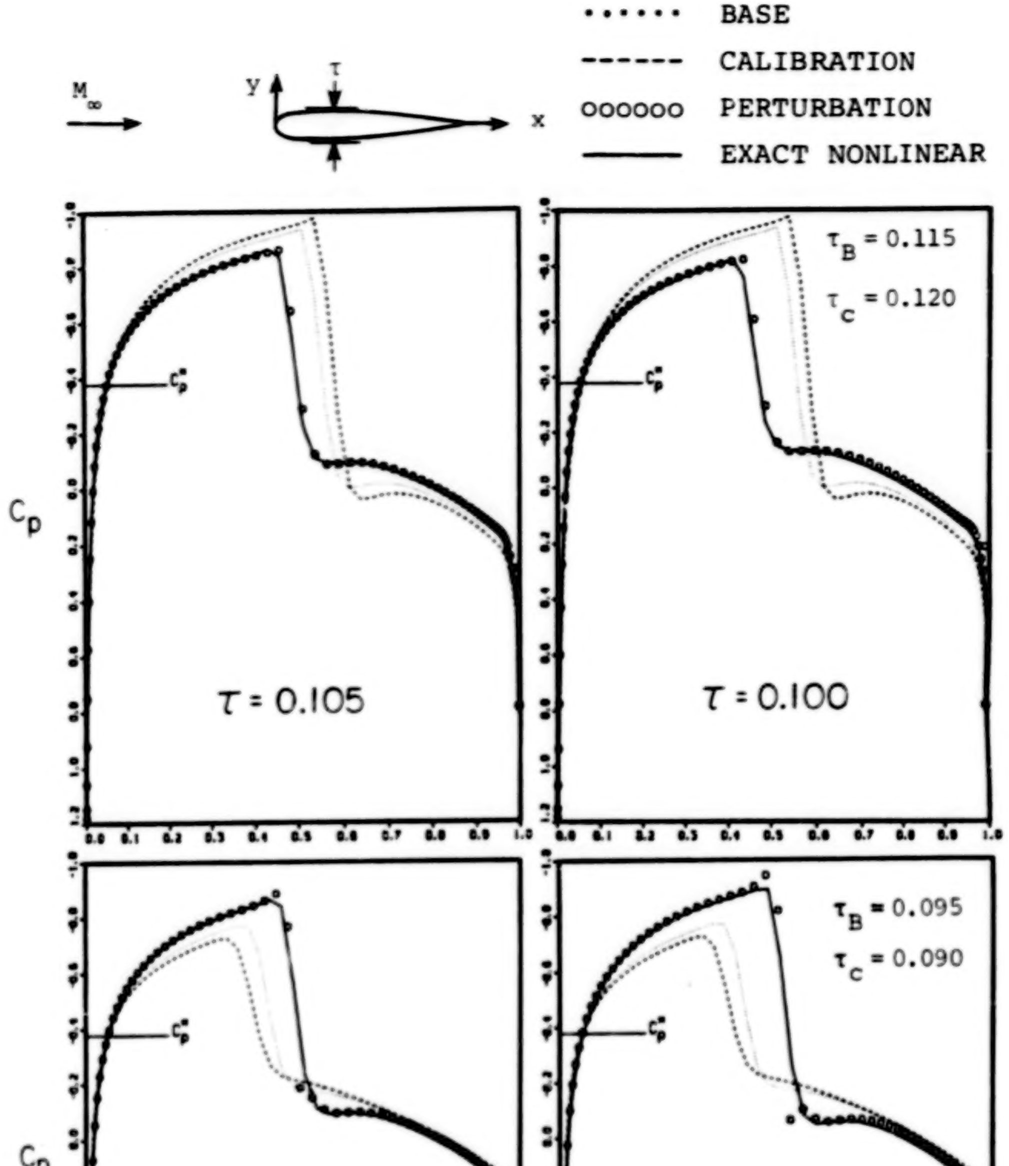


Figure 5.- Comparison of perturbation (O) and nonlinear (—) surface pressures for a thickness-ratio perturbation for an isolated NACA 00XX airfoil at M_∞ = 0.820 and α = 0° for extreme solution extrapolation

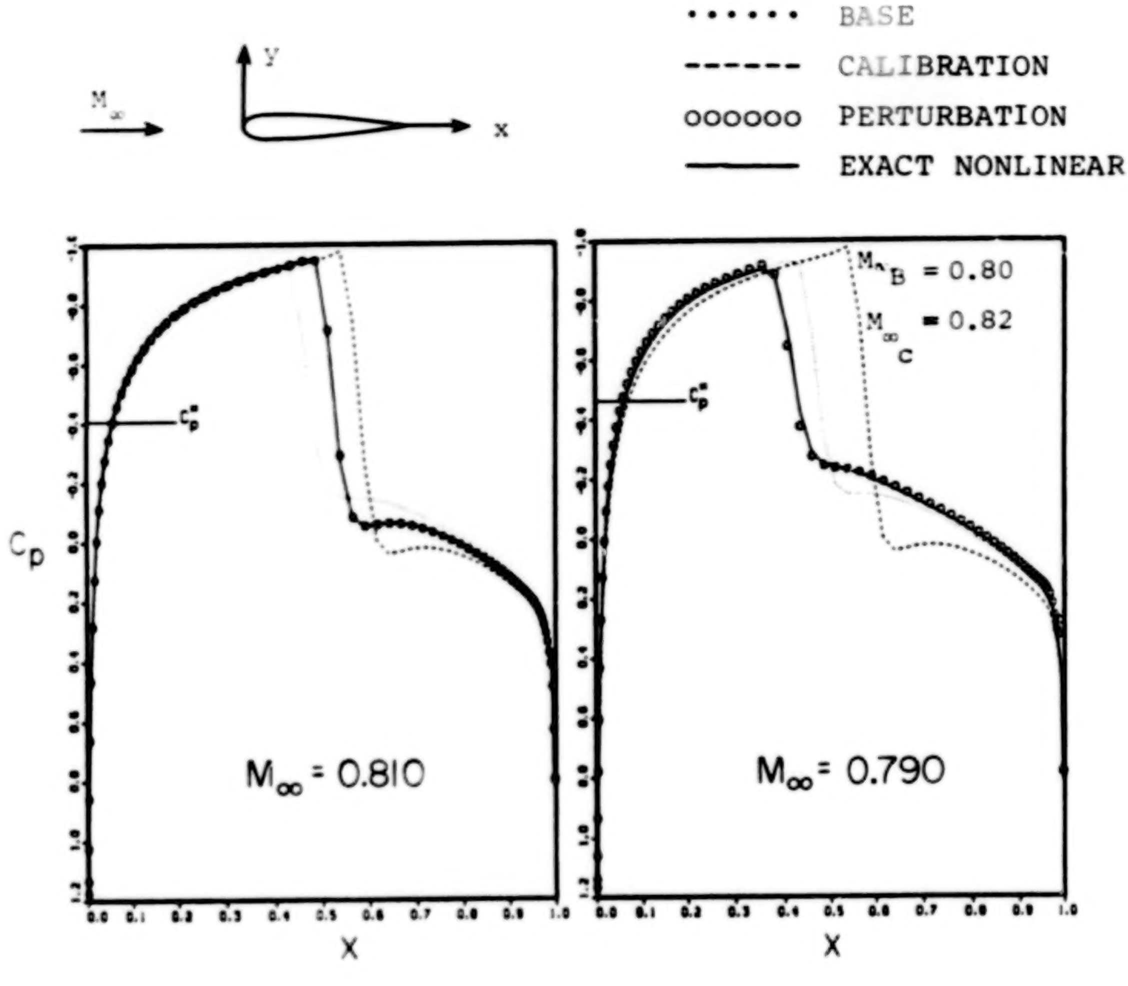


Figure 6.- Comparison of perturbation (O) and nonlinear (—) surface pressures for a Mach number perturbation of an isolated NACA 0012 airfoil at $\alpha = 0°$

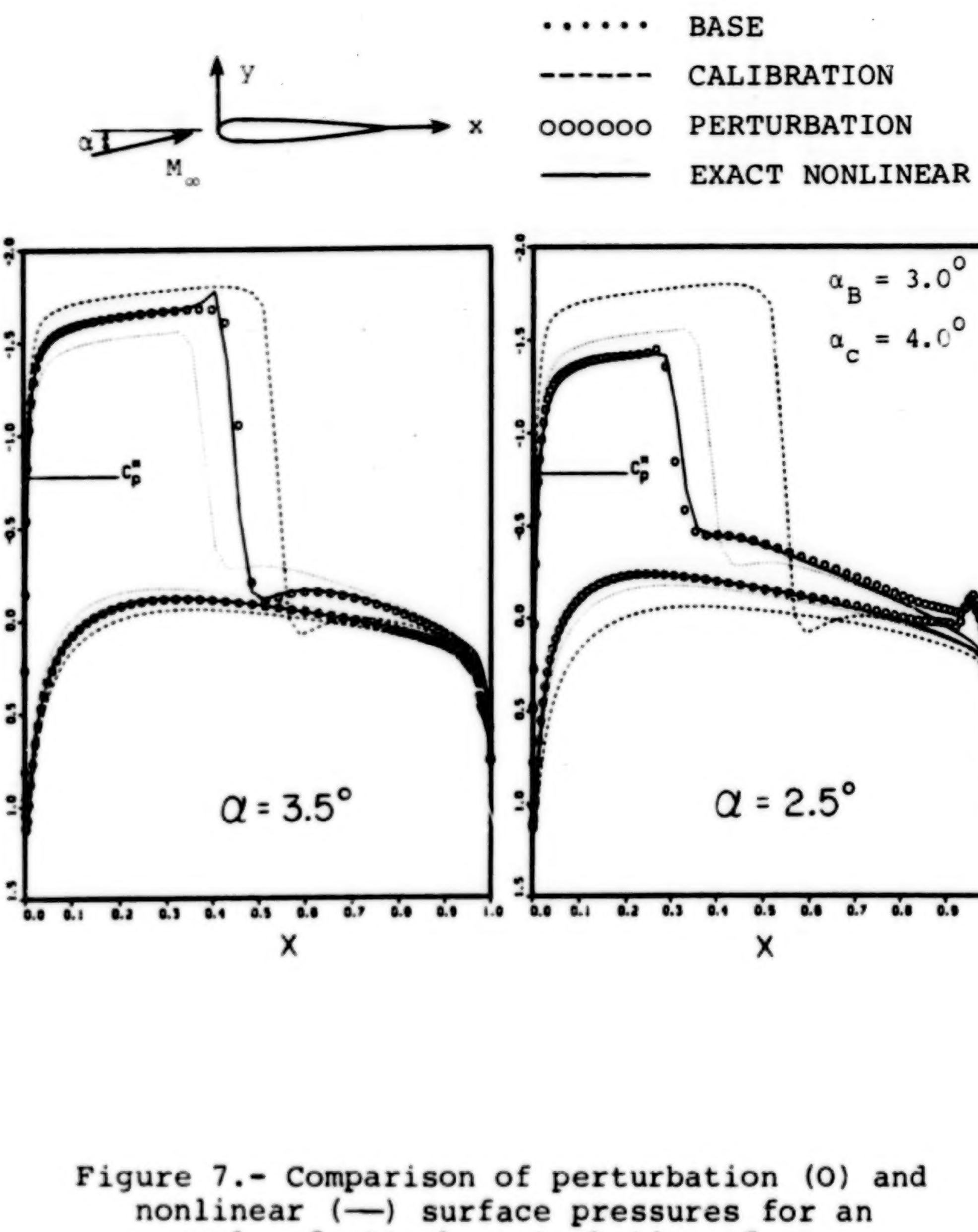


Figure 7.- Comparison of perturbation (O) and nonlinear (—) surface pressures for an angle-of-attack perturbation of an isolated NACA 0012 airfoil at $M_\infty = 0.70$

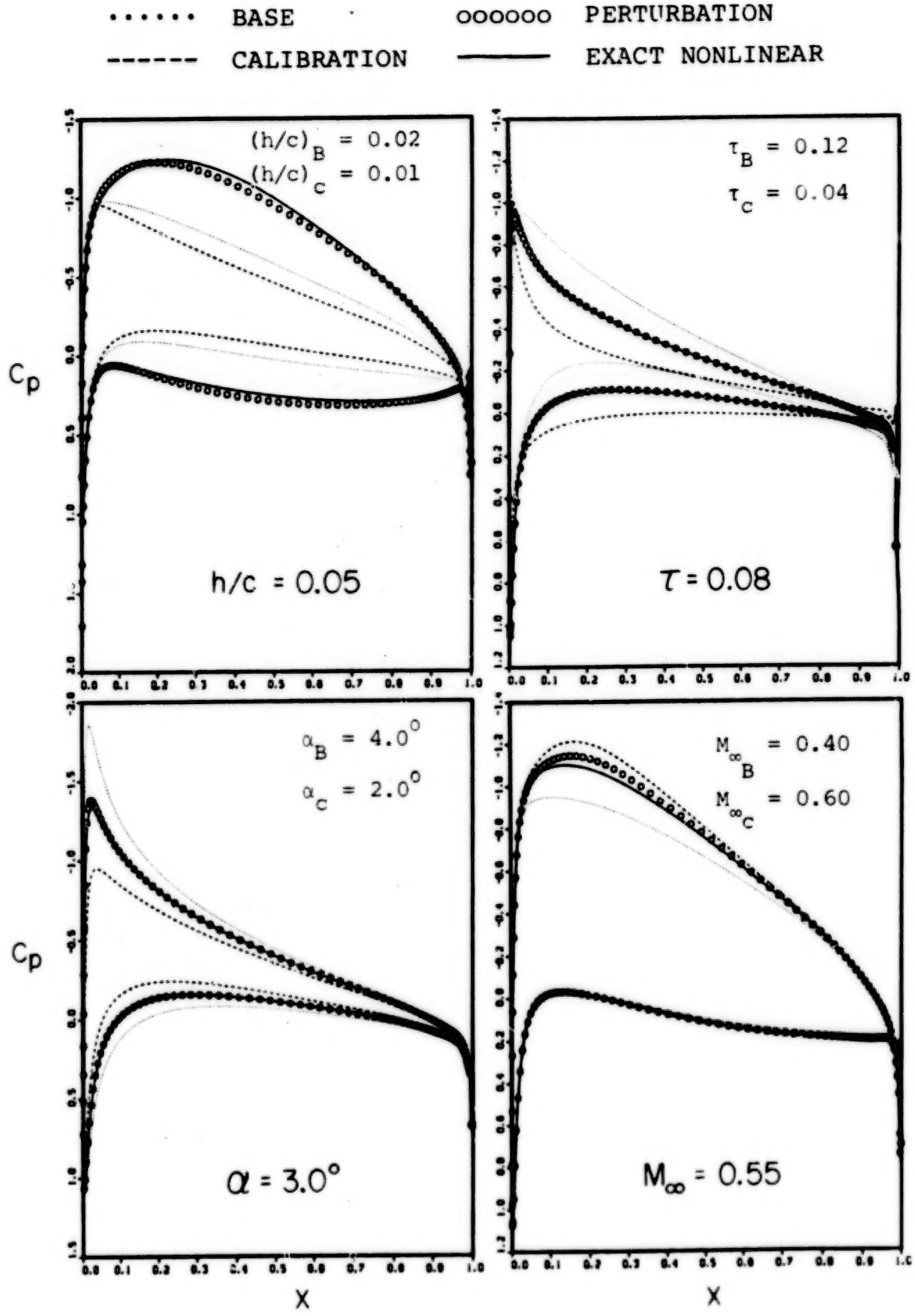


Figure 8.- Comparison of perturbation (O) and nonlinear (—) surface pressures for various geometry and flow parameter perturbations of isolated airfoils at subcritical speeds

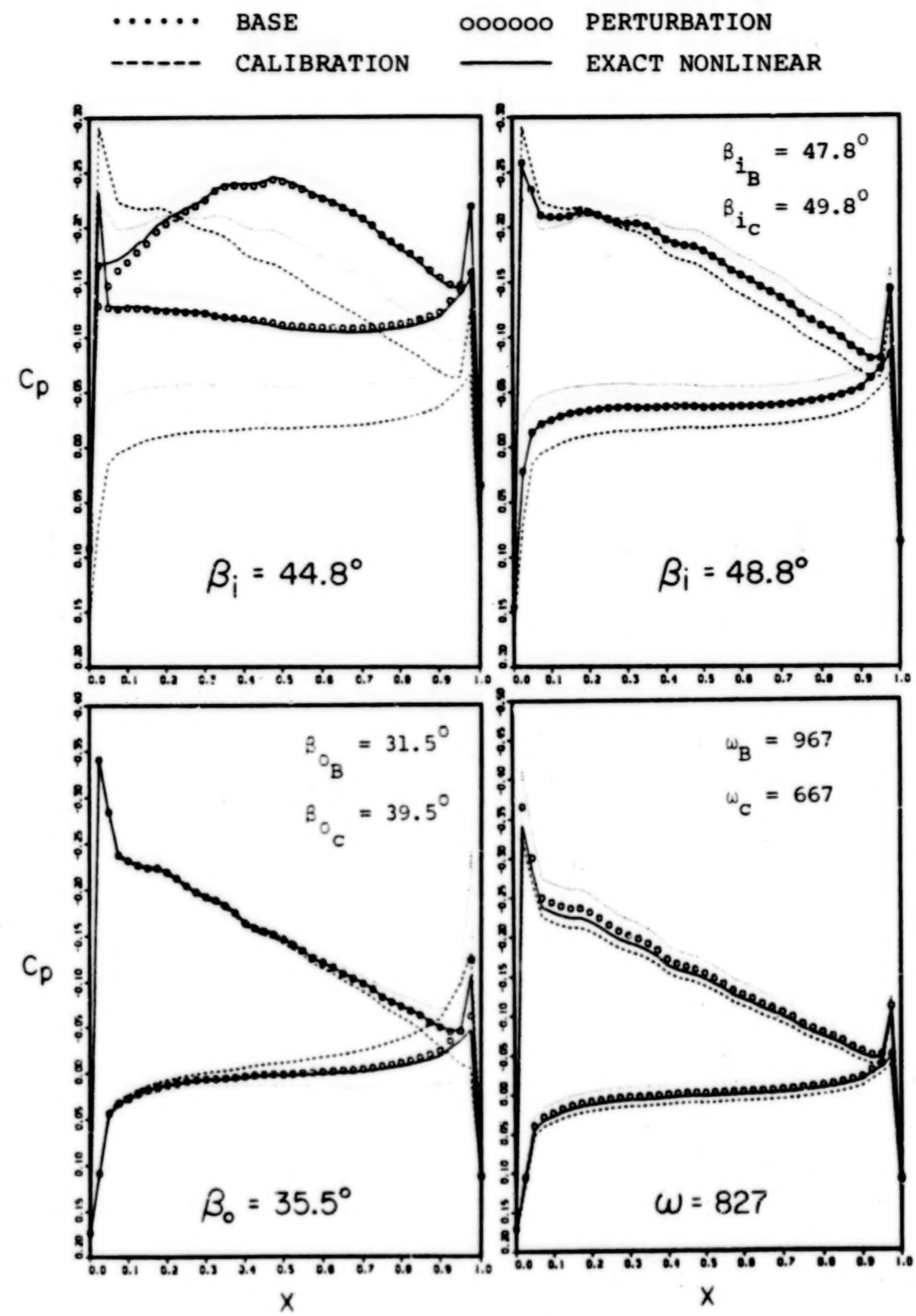


Figure 9.- Comparison of perturbation (O) and nonlinear (—) surface pressures for various flow parameter perturbations of a compressor cascade at subcritical speeds

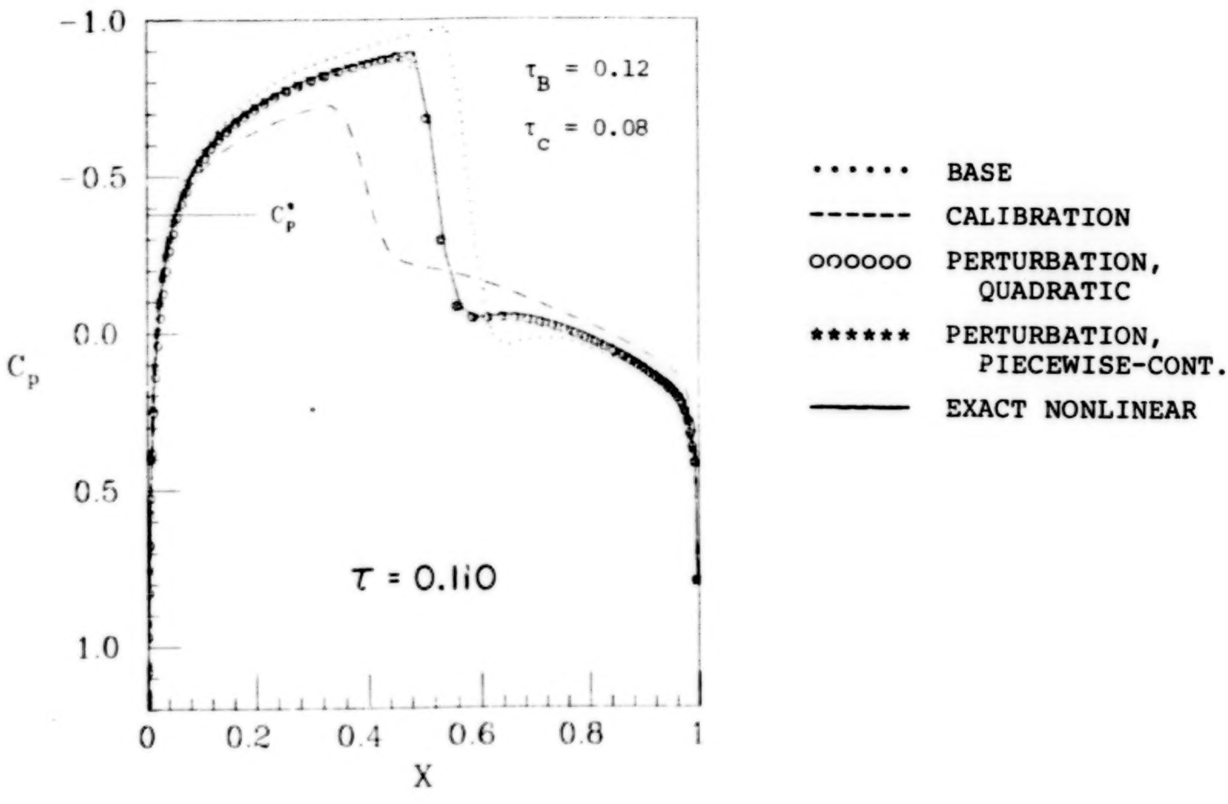


Figure 10.- Comparison of nonlinear (—) surface pressures with perturbation results using quadratic (O) and linear piecewise-continuous (*) straining functions for a thickness-ratio perturbation of an isolated NACA 00XX airfoil at $M_\infty = 0.820$ and $\alpha = 0°$

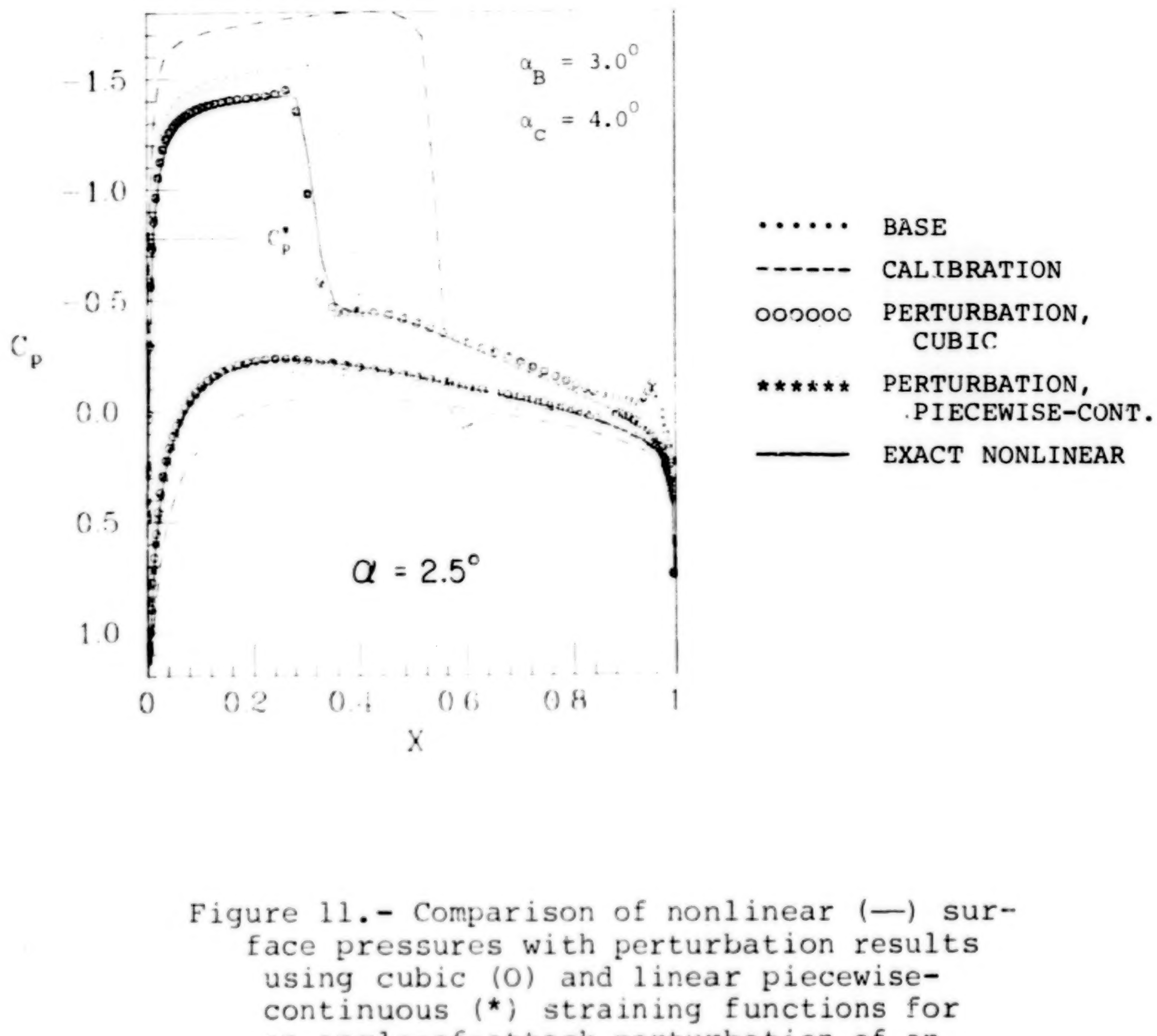


Figure 11.- Comparison of nonlinear (—) surface pressures with perturbation results using cubic (O) and linear piecewise-continuous (*) straining functions for an angle-of-attack perturbation of an isolated NACA 0012 airfoil at $M_\infty = 0.70$

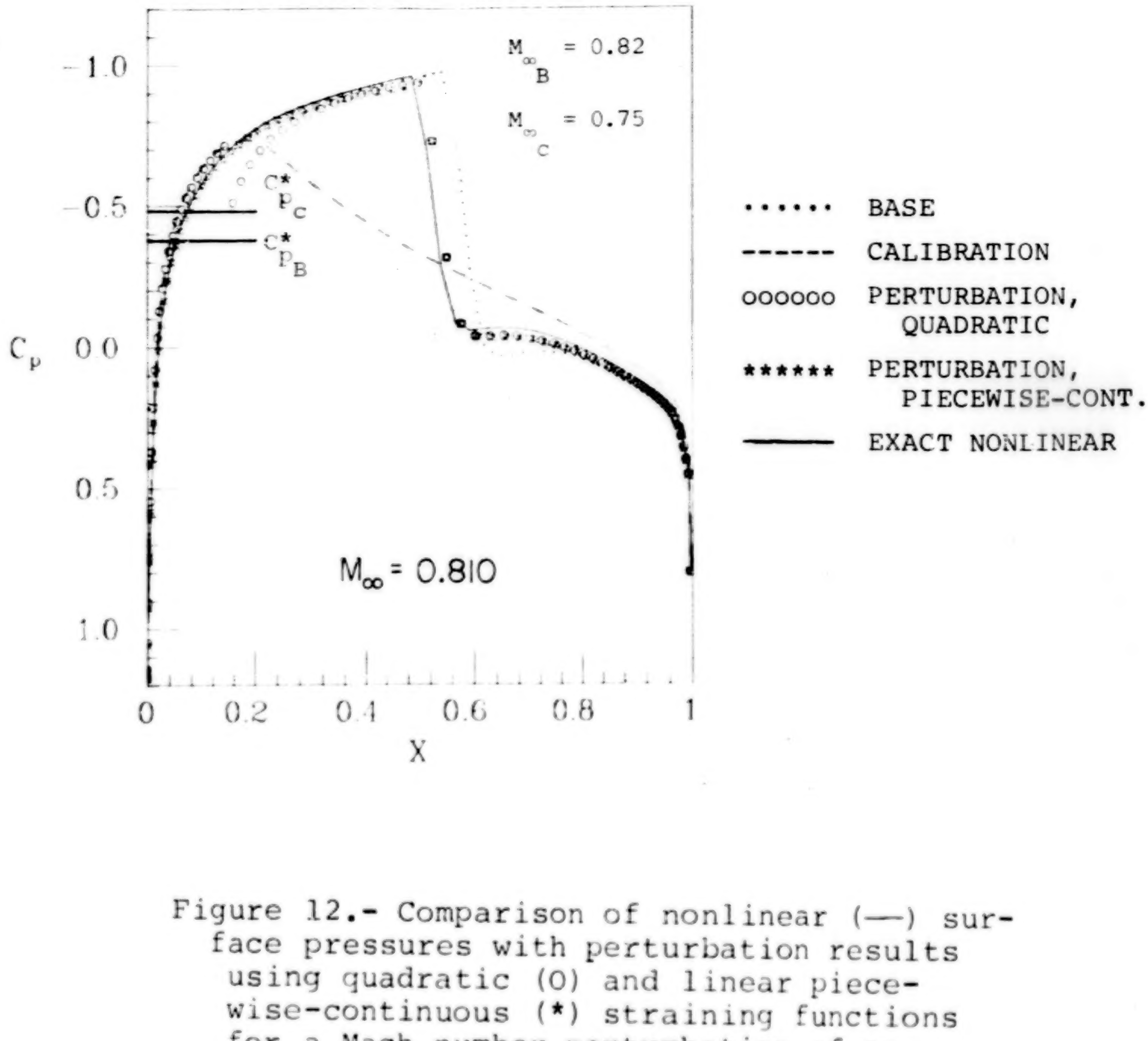


Figure 12.- Comparison of nonlinear (—) surface pressures with perturbation results using quadratic (O) and linear piecewise-continuous (*) straining functions for a Mach number perturbation of an isolated NACA 0012 airfoil at $\alpha = 0°$

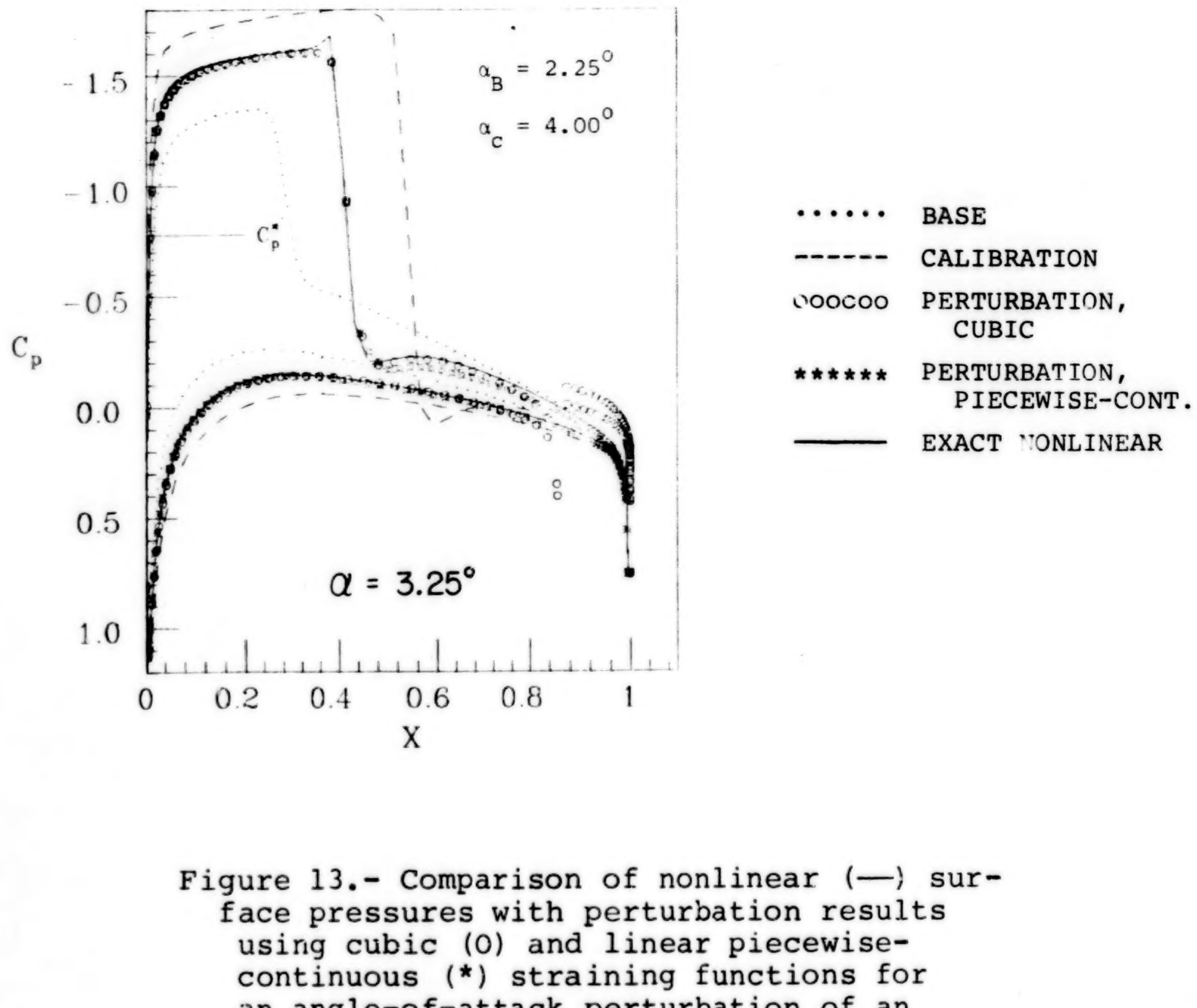


Figure 13.- Comparison of nonlinear (—) surface pressures with perturbation results using cubic (O) and linear piecewise-continuous (*) straining functions for an angle-of-attack perturbation of an isolated NACA 0012 airfoil at $M_\infty = 0.70$

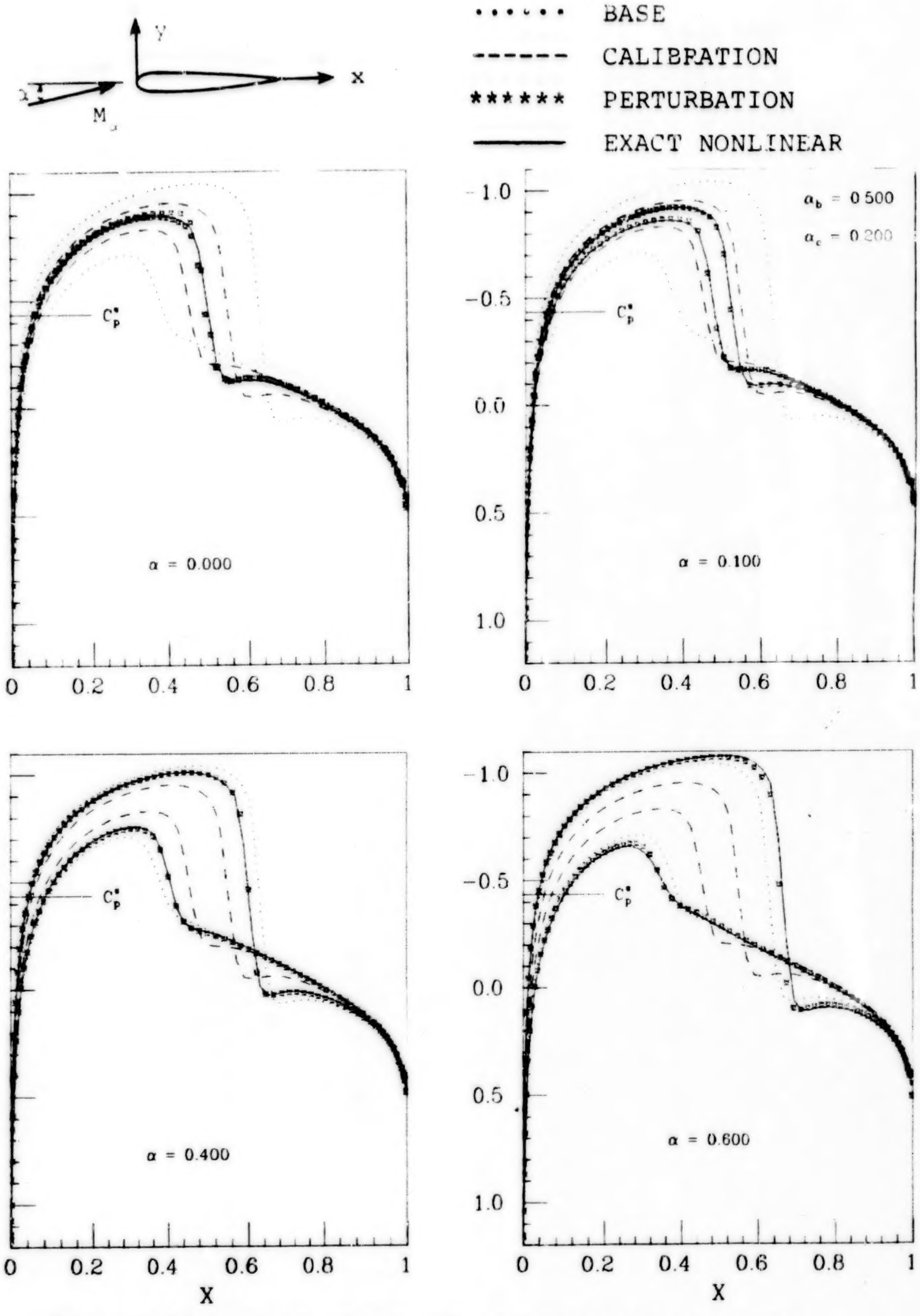


Figure 14.- Comparison of perturbation (*) and nonlinear (—) surface pressures for an angle-of-attack perturbation of an isolated NACA 0012 airfoil at $M_\infty = 0.80$ having multiple shocks

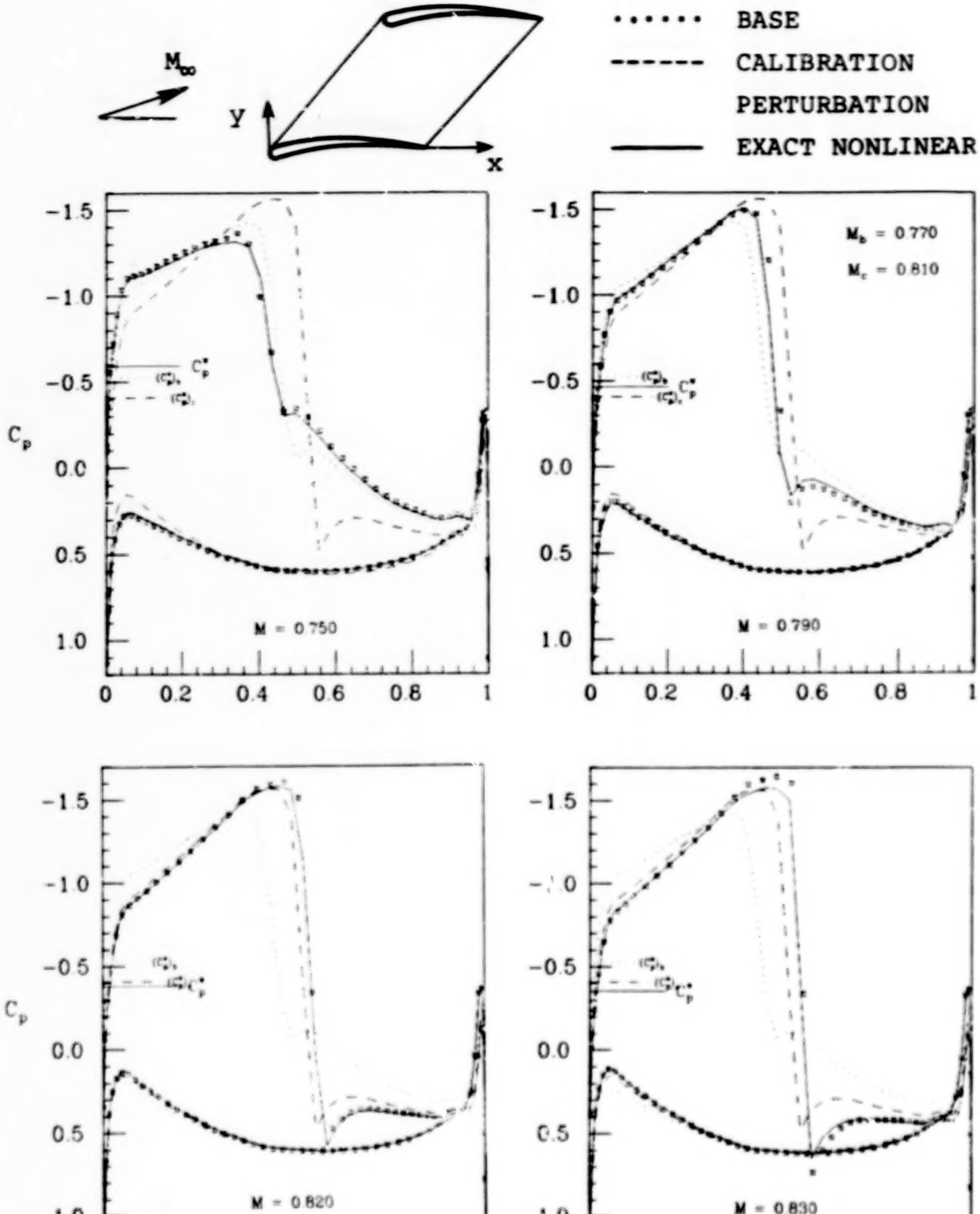


Figure 15.- Comparison of perturbation (*) and nonlinear (—) surface pressures for an oncoming Mach number perturbation of supercritical flow past a cascade of Jose Sanz blade profiles

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15. Supplementary Notes

Final report. Project Managers, William D. McNally and Aaron Snyder, Fluid Mechanics and Acoustics Division, NASA Lewis Research Center, Cleveland, Ohio 44135.

16. Abstract

An investigation was conducted to develop perturbation procedures and associated computational codes for determining nonlinear flow solutions, with the objective of establishing a method for minimizing computational requirements associated with parametric studies of transonic flows in turbomachines. The procedure that was developed and evaluated was found to be capable of determining highly accurate approximations to families of strongly nonlinear solutions which are either continuous or discontinuous, and which represent variations in some arbitrary parameter. Coordinate straining is employed to account for the movement of discontinuities and maxima of high-gradient regions due to the perturbation. Although simultaneous multiple-parameter perturbations can be treated, the development and results reported here are for the single-parameter perturbation problem. Flows past both isolated airfoils and compressor cascades involving a wide variety of flow and geometry parameter changes are reported. Attention is focused in particular on transonic flows which are strongly supercritical and exhibit large surface shock movement over the parametric range studied; and on subsonic flows which display large pressure variations in the stagnation and peak suction pressure regions. Comparisons with the corresponding 'exact' nonlinear solutions indicate a remarkable accuracy and range of validity of such a procedure. Computational time of the method, beyond the determination of the base solutions, is trivial.

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Steady flow	Strained coordinates	STAR Category 01
Turbomachinery	Blade-to-blade surface	

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